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# Can we reach the Zeptouniverse with rare $K$ and $B_{s,d}$ decays?

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**ABSTRACT:** The Large Hadron Collider (LHC) will directly probe distance scales as short as  $10^{-19}$  m, corresponding to energy scales at the level of a few TeV. In order to reach even higher resolutions before the advent of future high-energy colliders, it is necessary to consider indirect probes of New Physics (NP), a prime example being  $\Delta F = 2$  neutral meson mixing processes, which are sensitive to much shorter distance scales. However  $\Delta F = 2$  processes alone cannot tell us much about the structure of NP beyond the LHC scales. To identify for instance the presence of new quark flavour-changing dynamics of a left-handed (LH) or right-handed (RH) nature, complementary results from  $\Delta F = 1$  rare decay processes are vital. We therefore address the important question of whether NP could be seen up to energy scales as high as 200 TeV, corresponding to distances as small as  $\mathcal{O}(10^{-21})$  m — the *Zeptouniverse* — in rare  $K$  and  $B_{s,d}$  decays, subject to present  $\Delta F = 2$  constraints and perturbativity. We focus in particular on a heavy  $Z'$  gauge boson. If restricted to purely LH or RH  $Z'$  couplings to quarks, we find that rare  $K$  decays, in particular  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ , allow us to probe the Zeptouniverse. On the other hand rare  $B_s$  and  $B_d$  decays, which receive stronger  $\Delta F = 2$  constraints, allow us to reach about 15 TeV. Allowing for both LH and RH couplings a loosening of the  $\Delta F = 2$  constraints is possible, and we find that the maximal values of  $M_{Z'}$  at which NP effects could be found that are consistent with perturbative couplings are approximately 2000 TeV for  $K$  decays and 160 TeV for rare  $B_{s,d}$  decays. Because  $Z'$  exchanges in the  $B_{s,d} \rightarrow \mu^+ \mu^-$  rare decays are helicity suppressed, we also consider tree-level scalar exchanges for these decays, for which we find that scales close to 1000 TeV can be probed for the analogous pure and combined LH and RH scenarios. We further present a simple idea for an indirect determination of  $M_{Z'}$  that could be realised at the next linear  $e^+e^-$  or  $\mu^+\mu^-$  collider and with future precise flavour data.

**KEYWORDS:** Rare Decays, Beyond Standard Model, B-Physics, Kaon Physics**ARXIV EPRINT:** [1408.0728](https://arxiv.org/abs/1408.0728)

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# 1 Introduction

Through the recent discovery of the Higgs particle the Standard Model (SM) of strong and electroweak interactions is now complete, with the masses of all its particles being below 200 GeV, corresponding to scales above one Attometer ( $10^{-18}$  m). With the help of the Large Hadron Collider (LHC) the second half of this decade, together with the next decade, should allow us to probe directly the existence of other particles present in nature with masses up to a few TeV. Many models considered in the literature predict new gauge bosons, new fermions and new scalars in this mass range, but until now no clear signal of these new particles has been seen at the LHC. It is still possible that with the increased energy at the LHC new discoveries will be made in the coming years. But what if the lightest new particle in nature is in the multi-TeV range and out of the direct reach of the LHC?

The past successes of flavour physics in predicting new particles prior to their discovery may again help us in such a case, in particular in view of significant improvements on the precision of experiments and significant reduction of hadronic uncertainties through lattice QCD. But the question arises whether we will ever reach the energy scales as high as 200 TeV corresponding to short distances in the ballpark of  $10^{-21}$  m — the *Zeptouniverse* — in this manner and learn about the nature of New Physics (NP) at these very short distances.<sup>1</sup> The scale of 200 TeV is given here only as an example, and learning about NP at any scale above the LHC scale in this manner would be very important. Recent reviews on flavour physics beyond the SM can be found in [1, 2].

Some readers may ask why we are readdressing this question in view of the comprehensive analyses in the framework of effective theories in [3–5]. These analyses, which dealt dominantly with  $\Delta F = 2$  observables, have already shown that in the presence of left-right operators one could be in principle sensitive to scales as high as  $10^4$  TeV, or even higher scales. Here we would like to point out that the study of such processes alone will not really give us significant information about the particular nature of this NP. To this end also  $\Delta F = 1$  processes, in particular rare  $K$  and  $B_{s,d}$  decays, have to be considered. As left-right operators involving four quarks are not the driving force in these decays, which generally contain operators built out of one quark current and one lepton current, it is not evident that these decays can help us in reaching the Zeptouniverse even in the flavour precision era. In fact as will be evident from our analysis below, NP at scales well above 1000 TeV cannot be probed by rare meson decays.<sup>2</sup>

In this paper we address this question primarily in the context of one of the simplest extensions of the SM, a  $Z'$  model in which a heavy neutral gauge boson mediates FCNC processes in the quark sector at tree-level and has left-handed (LH) and/or right-handed (RH) couplings to quarks and leptons. This model has been studied recently for the general case in [16, 17] and in [18–20] in the context of 331 models. However, in these papers  $M_{Z'}$

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<sup>1</sup>We consider scales in the same ballpark, for example 50 TeV and 1000 TeV, which correspond respectively to 4 and 0.2 zeptometers and also belong to the Zeptouniverse.

<sup>2</sup>In principle this could be achieved in the future with the help of lepton flavour violating decays such as  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow 3e$ ,  $\mu \rightarrow e$  conversion in nuclei, and electric dipole moments [6–15].

has been chosen in the reach of the LHC, typically in the ballpark of 3 TeV. Here the philosophy will be to focus on the highest mass scales possibly accessible through flavour measurements. It is evident from [20] that in 331 models NP effects for  $M_{Z'} \geq 10$  TeV are too small to be measured in rare  $K$  and  $B_{s,d}$  decays even in the flavour precision era. On the other hand, as we will see, this is still possible in a general  $Z'$  model. References to other analyses in  $Z'$  models are collected in [1].

The  $Z'$  model that we will analyze is only one possible NP scenario and should thereby be considered as a useful concrete example in which our questions can be answered in explicit terms. It is nevertheless important to investigate whether other NP scenarios could also give sufficiently strong signals from very short distance scales so that they could be detected in future measurements. If fact we find that tree-level scalar exchanges could also give us informations about these very short scales through  $B_{s,d} \rightarrow \mu^+ \mu^-$  decays.

Our paper is organised as follows. In section 2 we outline the strategy for finding the maximal possible resolution of short distance scales with the help of rare meson decays. This depends on the *maximal value* of the  $Z'$  couplings to fermions that are allowed by perturbativity and present experimental constraints. It also depends on the *minimal* deviations from SM expectations that in the flavour precision era could be considered as a clear signal of NP. In section 3 we perform the analysis for  $Z'$  scenarios with only LH or only RH flavour violating couplings to quarks. In section 4 the case of  $Z'$  with LH and RH flavour violating couplings to quarks is analysed. In section 5 we repeat the analysis of previous sections for tree-level (pseudo-)scalar contributions restricting the discussion to the decays  $B_{s,d} \rightarrow \mu^+ \mu^-$ . In section 6 we discuss briefly other NP scenarios. In section 7 we present a simple idea for a rough indirect determination of  $M_{Z'}$  by means of the next linear  $e^+e^-$  or  $\mu^+\mu^-$  collider and flavour data. We conclude in section 8.

## 2 Setup and strategy

The virtue of the  $Z'$  scenarios is the paucity of their parameters that enter all flavour observables in a given meson system, which should be contrasted with most NP scenarios outside the Minimal Flavour Violation (MFV) framework. Indeed, the  $\Delta F = 2$  and  $\Delta F = 1$  transitions in the  $K$ ,  $B_d$  and  $B_s$  systems are fully described by the following ratios of the  $Z'$  couplings to SM fermions over its mass  $M_{Z'}$ ,

$$\Delta_{L,R}^{sd}/M_{Z'}, \quad \Delta_{L,R}^{bd}/M_{Z'}, \quad \Delta_{L,R}^{bs}/M_{Z'}, \quad (2.1)$$

and

$$\Delta_L^{\nu\bar{\nu}}/M_{Z'}, \quad \Delta_A^{\mu\bar{\mu}}/M_{Z'}, \quad \Delta_V^{\mu\bar{\mu}} = 2\Delta_L^{\nu\bar{\nu}} + \Delta_A^{\mu\bar{\mu}}, \quad (2.2)$$

where the last formula follows from the  $SU(2)_L$  symmetry relation  $\Delta_L^{\nu\bar{\nu}} = \Delta_L^{\mu\bar{\mu}}$ . These couplings are defined as in [16, 17] through

$$\mathcal{L}_{\text{FCNC}}^{\text{quarks}} = \left[ \bar{q}_i \gamma_\mu P_L q_j \Delta_L^{ij} + \bar{q}_i \gamma_\mu P_R q_j \Delta_R^{ij} + h.c. \right] Z'^\mu, \quad (2.3)$$

with  $i, j = d, s, b$  and  $i \neq j$  throughout the rest of the paper. The analogous definition applies to the lepton sector where only flavour conserving couplings are considered,

$$\mathcal{L}^{\text{leptons}} = [\bar{\mu} \gamma_\mu P_L \mu \Delta_L^{\mu\bar{\mu}} + \bar{\mu} \gamma_\mu P_R \mu \Delta_R^{\mu\bar{\mu}} + \bar{\nu} \gamma_\mu P_L \Delta_L^{\nu\bar{\nu}}] Z'^\mu. \quad (2.4)$$

We recall that the couplings  $\Delta_{A,V}^{\mu\bar{\mu}}$  are defined as

$$\Delta_V^{\mu\bar{\mu}} = \Delta_R^{\mu\bar{\mu}} + \Delta_L^{\mu\bar{\mu}}, \quad \Delta_A^{\mu\bar{\mu}} = \Delta_R^{\mu\bar{\mu}} - \Delta_L^{\mu\bar{\mu}}. \quad (2.5)$$

Other definitions and normalisation of couplings can be found in [16]. The quark couplings are in general complex whereas the leptonic ones are assumed to be real.

It is evident from these expressions that in order to find out the maximal value of  $M_{Z'}$  for which measurable NP effects in  $\Delta F = 2$  and  $\Delta F = 1$  exist one has to know the maximal values of the couplings  $\Delta_{L,R}^{ij}$  and  $\Delta_{L,R}^{\mu\bar{\mu}}$  allowed by perturbativity. From the  $\Delta F = 2$  analyses in [3–5] it follows that by choosing these couplings to be  $\mathcal{O}(1)$  the lower bound on the scale of new physics  $\Lambda_{\text{NP}}$  could be in the range of  $10^5$  TeV for the case of  $K^0 - \bar{K}^0$  mixing. On the other hand, choosing sufficiently small couplings by means of a suitable flavour symmetry it is possible to suppress the FCNCs related to NP with the NP scale  $\Lambda_{\text{NP}}$  in the ballpark of a few TeV [21–28].

In view of the fact that flavour physics in the rest of this decade and in the next decade will be dominated by new precise measurements of rare  $K$  and rare  $B_{s,d}$  decays and not  $\Delta F = 2$  transitions, our strategy will differ from the one in [3–5]. We will assume that future measurements will be precise enough to identify conclusively the presence of NP in rare decays when the deviations from SM predictions for various branching ratios will be larger than 10–30% of the SM branching ratio. The precise value of the detectable deviation will depend on the decay considered and will be smaller for the ones with smaller experimental, hadronic and parametric uncertainties. We will be more specific about this in the next section. The framework considered here goes beyond MFV, where even for  $\Lambda_{\text{NP}}$  in the ballpark of a few TeV only moderate departures from the SM in  $\Delta F = 1$  observables are predicted. A model independent analysis of  $b \rightarrow s$  transitions in this framework can be found in [29] and in a recent review in [30].

In order to proceed we have to make assumptions about the size of the couplings involved. There is in general a lot of freedom here, but as we are searching for the maximal values of  $M_{Z'}$  which could still provide measurable NP effects in rare meson decays, we will choose maximal couplings that are consistent with perturbativity. Subsequently we will check whether such couplings are also consistent with  $\Delta F = 2$  constraints for a given  $M_{Z'}$ . An estimate of the perturbativity upper bound on  $\Delta_{L,R}^{sd}$  was made in [31], in the context of a study of the isospin amplitude  $A_0$  in  $K \rightarrow \pi\pi$  decays, by considering the loop expansion parameter

$$L = N_c \left( \frac{\Delta_{L,R}^{sd}}{4\pi} \right)^2, \quad (2.6)$$

where  $N_c = 3$  is the number of colours. For  $\Delta_{L,R}^{sd} = 3.0$  we find  $L = 0.17$ , a coupling strength that is certainly allowed. The same estimate can be made for other LH and RH

Observable	2014	2019	2024	2030
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(17.3^{+11.5}_{-10.5}) \times 10^{-11}$ [32]	10% [33]	5% [34]	
$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$	$< 2.6 \times 10^{-8}$ (90% CL) [35]		5% [34]	
$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$	$< 1.3 \times 10^{-5}$ (90% CL) [36]		30% [37]	
$\mathcal{B}(B_d^0 \rightarrow K^{*0} \nu \bar{\nu})$	$< 5.5 \times 10^{-5}$ (90% CL) [38]		35% [37]	
$\bar{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)$	$(2.9 \pm 0.7) \times 10^{-9}$ [39–41]	15% [42, 43]	12% [42]	10–12% [42, 43]
$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$	$(3.6^{+1.6}_{-1.4}) \times 10^{-10}$ † [39–41]	66% [42]	45% [42]	18% [42]
$\mathcal{B}(B_d \rightarrow \mu^+ \mu^-) / \bar{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)$		71% [42]	47% [42]	21–35% [42, 43]

**Table 1.** The current best experimental measurements (2014) together with the precision expected in 5, 10 and 15 years for the rare decay observables studied in this paper. The percentages are relative to SM predictions. †The statistical significance of this measurement is less than  $3\sigma$  i.e. there is still no *evidence* for this process.  $\bar{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)$  denotes the corrected branching ratio as defined in appendix B.6.

couplings considered by us. However, as we will see below, the correlation of  $\Delta F = 1$  and  $\Delta F = 2$  processes in the case of  $Z'$  exchange, derived in [16], will give some additional insight on the allowed size of the quark couplings and will generally not allow us to reach the perturbativity bounds on quark couplings. On the other hand, large values of the leptonic couplings  $\Delta_L^{\nu\bar{\nu}}$  and  $\Delta_{V,A}^{\mu\bar{\mu}}$  at the perturbativity upper bound will give an estimate of the maximal  $M_{Z'}$  for which measurable effects in rare  $K$  and  $B_{s,d}$  decays could be obtained.

In the case of a  $U(1)$  gauge symmetry with large gauge couplings at a given scale it is difficult to avoid a Landau pole at still higher scales. However, for the coupling values used in our paper, this happens at much higher scales than  $M_{Z'}$ . Moreover, if  $Z'$  is associated with a non-abelian gauge symmetry that is asymptotically free this problem does not exist.

**Projections for the coming years.** Clearly, the outcome of our strategy depends sensitively on the precision of future measurements and the reduction of hadronic and CKM uncertainties. In table 1 we give the precision expected in the next 5, 10 and 15 years for the rare decay observables that we study in this paper. In table 2 we do the same for the lattice and CKM matrix parameters that contribute with sizeable errors in our numerical analysis. We also list the current experimental precision for these quantities. The chosen years of 2019, 2024 and 2030 correspond approximately to the integrated luminosity milestones of the relevant experiments. For Belle-II the years 2019 and 2024 correspond to  $5 \text{ ab}^{-1}$  and  $50 \text{ ab}^{-1}$ , respectively. For LHCb the years 2019, 2024 and 2030 correspond to  $6 \text{ fb}^{-1}$ ,  $15 \text{ fb}^{-1}$  and  $50 \text{ fb}^{-1}$ , respectively. For CMS the years 2018, 2024 and 2030 correspond to  $100 \text{ fb}^{-1}$ ,  $300 \text{ fb}^{-1}$  and  $3000 \text{ fb}^{-1}$ , respectively. Needless to say all these projections can change in the future, yet the collected numbers show that the coming years indeed deserve the label of the *flavour precision era*. In view of these prospects we will keep in mind throughout this paper that NP effects that are at least as large as 10–30% of the SM branching ratios could one day be resolved in rare meson decays. We will be more explicit about this in the next section.

	2014	2019	2024	2030
$F_{B_s}$	$(227.7 \pm 4.5) \text{ MeV}$ [44]	$< 1\%$ [45]		
$F_{B_d}$	$(190.5 \pm 4.2) \text{ MeV}$ [44]	$< 1\%$ [45]		
$F_{B_s} \sqrt{\hat{B}_{B_s}}$	$(266 \pm 18) \text{ MeV}$ [44]	$2.5\%$ [45]	$< 1\%$ [46]	
$F_{B_d} \sqrt{\hat{B}_{B_d}}$	$(216 \pm 15) \text{ MeV}$ [44]	$2.5\%$ [45]	$< 1\%$ [46]	
$\hat{B}_K$	$0.766 \pm 0.010$ [44]	$< 1\%$ [45]		
$ V_{ub} _{\text{incl}}$	$(4.40 \pm 0.25) \times 10^{-3}$ [44]	$5\%$ [37]	$3\%$ [37]	
$ V_{ub} _{\text{excl}}$	$(3.42 \pm 0.31) \times 10^{-3}$ [44]	$12\% \text{ }^{\dagger\dagger}$ [37]	$5\% \text{ }^{\dagger\dagger}$ [37]	
$ V_{cb} _{\text{incl}}$	$(42.4 \pm 0.9) \times 10^{-3}$ [47]	$1\%$ [48]	$< 1\%$ [48]	
$ V_{cb} _{\text{excl}}$	$(39.4 \pm 0.6) \times 10^{-3}$ [44]	$1\%$ [48]	$< 1\%$ [48]	
$\gamma$	$(70.1 \pm 7.1)^\circ \text{ }^{\dagger}$ [49]	$6\%$ [37]	$1.5\%$ [37]	$1.3\%$ [43]
$\phi_d^{\text{SM}} = 2\beta$	$(43.0^{+1.6}_{-1.4})^\circ$ [50]	$\sim 1^\circ \text{ }^{\dagger}$ [51, 52]		
$\phi_s^{\text{SM}} = -2\beta_s$	$(0 \pm 4)^\circ$ [50]	$1.4^\circ$ [43]	$\sim 1^\circ \text{ }^{\dagger}$ [53]	

**Table 2.** Current best determinations and future forecasts for the precision of lattice and CKM matrix parameters that contribute with sizeable errors in our numerical analysis. <sup>†</sup>Combined fit from charmed B decay modes. <sup>††</sup>These predictions assume dominant lattice errors. <sup>‡</sup>At this precision the theoretical uncertainty due to penguin pollution in the dominant decay modes used to extract these phases starts to dominate.

### 3 Left-handed and right-handed $Z'$ scenarios

#### 3.1 Left-handed scenario

It will be useful to begin our analysis with the case of  $Z'$  having only LH flavour violating couplings to quarks  $\Delta_L^{ij}$ . In this scenario NP effects from  $Z'$  can be compactly summarised through the flavour non-universal shifts in the basic functions  $X$ ,  $Y$  and  $S$ , as defined in [1, 16, 67], which are flavour universal in the SM:

$$X_L(M) = X^{\text{SM}} + \Delta X_L(M), \quad (3.1)$$

$$Y_A(M) = Y^{\text{SM}} + \Delta Y_A(M), \quad (3.2)$$

$$S(M) = S^{\text{SM}} + \Delta S(M), \quad (3.3)$$

with  $M = K, B_d, B_s$ .  $X_L(M)$  and  $Y_A(M)$  enter the amplitudes for decays with  $\nu\bar{\nu}$  and  $\mu\bar{\mu}$  final states, respectively;  $S(M)$  enters  $\Delta F = 2$  transitions. We recall that the functions  $X^{\text{SM}}$ ,  $Y^{\text{SM}}$  and  $S^{\text{SM}}$  enter the top quark contributions to the corresponding amplitudes in the SM. We suppressed here for simplicity the functions related to vector ( $V$ ) couplings. We will return to them later on.

In what follows we will concentrate our discussion mainly on the functions  $\Delta X_L(M)$ , since in the left-handed scenario (l.h.s.)  $\Delta Y_A(M)$  are given by [16]

$$\Delta Y_A(K) = \Delta X_L(K) \frac{\Delta_A^{\mu\bar{\mu}}}{\Delta_L^{\nu\bar{\nu}}}, \quad \Delta Y_A(B_q) = \Delta X_L(B_q) \frac{\Delta_A^{\mu\bar{\mu}}}{\Delta_L^{\nu\bar{\nu}}}, \quad (3.4)$$

as follows from the definitions of these functions given in appendix A.



The fundamental equations for the next steps of our analysis are the correlations in the l.h.s. between  $\Delta X(M)$  and  $\Delta S(M)$  derived in [16]. Rewriting them in a form suitable for our applications we find

$$\frac{\Delta X_L(K)}{\sqrt{\Delta S(K)}} = \frac{\Delta X_L(B_q)}{\sqrt{\Delta S(B_q)^*}} = \frac{\Delta_L^{\nu\bar{\nu}}}{2M_{Z'}g_{\text{SM}}\sqrt{\tilde{r}}} = 0.25 \left[ \frac{\Delta_L^{\nu\bar{\nu}}}{3.0} \right] \left[ \frac{15 \text{ TeV}}{M_{Z'}} \right], \quad (3.5)$$

where  $\tilde{r}$  is a QCD correction which depends on the  $Z'$  mass [16] ( $\tilde{r} \approx 0.90$  for  $M_{Z'} = 50 \text{ TeV}$ , but its dependence on  $M_{Z'}$  is very weak), and

$$g_{\text{SM}}^2 = 4 \frac{M_W^2 G_F^2}{2\pi^2} = 1.78137 \times 10^{-7} \text{ GeV}^{-2}, \quad (3.6)$$

where  $G_F$  is the Fermi constant.

Now comes an important observation: in the limit where the  $Z'$  coupling  $\Delta_L^{sd}$  is approximately real and the  $\varepsilon_K$  constraint is easily satisfied, the allowed range for  $\Delta S(K)$  can be much larger than the ones for  $\Delta S(B_q)$  even if the ratios in (3.5) are flavour universal. Indeed the  $\Delta S(B_q)$  are directly constrained by the  $B_q^0 - \bar{B}_q^0$  mass differences  $\Delta M_q$  because the function  $S_{\text{SM}}$  enters the top quark contribution to  $\Delta M_q$ , which is by far dominant in the SM. On the other hand  $\Delta M_K$  is dominated in the SM by charm quark contribution and the function  $S$  is multiplied there by small CKM factors. Consequently, the shift  $\Delta S(K)$  is allowed to be much larger than the shifts in  $\Delta S(B_q)$ , with interesting consequences for rare  $K$  decays as discussed below. Of course this assumes that the SM gives a good description of the experimental values of  $\varepsilon_K$  and  $\varepsilon'/\varepsilon$ . We will relax this assumption later.

Let us first illustrate the case of  $\Delta S(B_s)$  in the simplified scenario where  $\Delta_L^{bs}$  is real, in accordance with the small CP violation observed in the  $B_s$  system. Assuming then that a NP contribution to  $\Delta M_s$  at the level of 15% is still allowed, the result of taking into account all the experimental and hadronic uncertainties implies that only  $|\Delta S(B_s)| \leq 0.36$  is allowed by present data. This gives

$$|\Delta X_L(B_q)| \leq 0.16 \sqrt{\frac{|\Delta S(B_q)|}{0.36}} \left[ \frac{\Delta_L^{\nu\bar{\nu}}}{3.0} \right] \left[ \frac{15 \text{ TeV}}{M_{Z'}} \right]. \quad (3.7)$$

Since  $X^{\text{SM}} \approx 1.46$ , the shift  $|\Delta X_L(B_q)| = 0.16$  amounts to about 11% at the level of the amplitude and 22% for the branching ratios. Such NP effects could in principle one day be measured in  $b \rightarrow s\nu\bar{\nu}$  transitions such as  $B_d \rightarrow K(K^*)\nu\bar{\nu}$  and  $B \rightarrow X_s\nu\bar{\nu}$ , and can still be increased by increasing slightly  $\Delta_L^{\nu\bar{\nu}}$  or lowering  $M_{Z'}$ . However, this analysis shows that with the help of a  $Z'$  with only LH couplings one cannot reach the Zeptouniverse using  $B_s$  decays, although distance scales in the ballpark of  $10^{-20}\text{m}$ , corresponding to 15 TeV, could be resolved. A similar analysis can be performed for the function  $Y_A(B_s)$  relevant for  $B_s \rightarrow \mu^+\mu^-$ : as  $Y^{\text{SM}} \approx 0.96$ , a shift of  $|\Delta Y_A(B_s)| = 0.16$  results in a 33% modification in the branching ratio.

For  $B_d$  the discussion is complicated by the significant phase of  $V_{td}$ . Because  $|V_{td}| \approx 0.25|V_{ts}|$ , at first sight one may expect the shortest distance scales that can be resolved with rare  $B_d$  decays to be about two times higher than the ones for  $B_s$ . But, as seen in (3.5) for fixed lepton couplings, only  $M_{Z'}$  and the  $\Delta F = 2$  constraints on  $S$  determine



$ \epsilon_K  = 2.228(11) \times 10^{-3}$	[54]	$\alpha_s(M_Z) = 0.1185(6)$	[55]
$\Delta M_K = 0.5292(9) \times 10^{-2} \text{ ps}^{-1}$	[54]	$m_s(2 \text{ GeV}) = 93.8(24) \text{ MeV}$	[44]
$\Delta M_d = 0.507(4) \text{ ps}^{-1}$	[56]	$m_c(m_c) = 1.279(13) \text{ GeV}$	[57]
$\Delta M_s = 17.72(4) \text{ ps}^{-1}$	[56]	$m_b(m_b) = 4.19^{+0.18}_{-0.06} \text{ GeV}$	[54]
$ V_{us}  = 0.2252(9)$	[56]	$m_t(m_t) = 163(1) \text{ GeV}$	[58, 59]
$\Delta\Gamma_s/\Gamma_s = 0.123(17)$	[56]	$F_K = 156.1(11) \text{ MeV}$	[58]
$m_K = 497.614(24) \text{ MeV}$	[54]	$F_{B^+} = 185(3) \text{ MeV}$	[60]
$m_{B_d} = m_{B^+} = 5279.2(2) \text{ MeV}$	[55]	$\kappa_\epsilon = 0.94(2)$	[61, 62]
$m_{B_s} = 5366.8(2) \text{ MeV}$	[55]	$\eta_{cc} = 1.87(76)$	[63]
$\tau_{B^\pm} = 1.642(8) \text{ ps}$	[56]	$\eta_{tt} = 0.5765(65)$	[64]
$\tau_{B_d} = 1.519(7) \text{ ps}$	[56]	$\eta_{ct} = 0.496(47)$	[65]
$\tau_{B_s} = 1.509(11) \text{ ps}$	[56]	$\eta_B = 0.55(1)$	[64, 66]

**Table 3.** Values of other experimental and theoretical quantities used as input parameters. For future updates see PDG [55], FLAG [44] and HFAG [56].

the maximal size of  $\Delta F = 1$  effects, independently of the CKM matrix elements. Similar effects to the ones allowed for rare  $B_s$  decays are therefore also expected for rare  $B_d$  decays in l.h.s., for the same values of  $M_{Z'}$ . Slightly lower scales than 15 TeV can however be reached in this case, as is shown in our analysis below, because of the lower experimental precision expected for rare  $B_d$  decays (see table 1).

The prospects for the observation of NP in  $B_{d,s} \rightarrow \mu^+ \mu^-$  are shown in figure 1 for the following benchmark scenario:

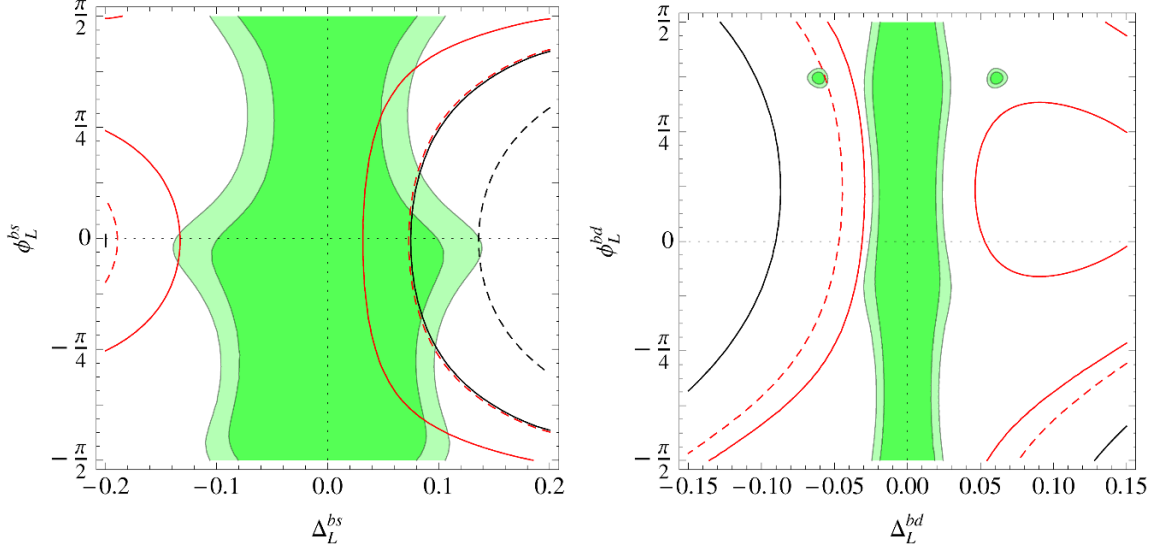
- $M_{Z'} = 15 \text{ TeV}$ , which corresponds approximately to the highest accessible scale, and  $\Delta_A^{\mu\bar{\mu}} = -3$ ; the negative sign of  $\Delta_A^{\mu\bar{\mu}}$  is compatible with (2.2) and perturbativity for  $\Delta_L^{\nu\bar{\nu}} = 3.0$  (to be discussed in section 3.3, see in particular (3.16)).

Virtually identical results are obtained for  $M_{Z'} = 5 \text{ TeV}$ , which is in the reach of direct detection at the LHC [68, 69], and  $\Delta_A^{\mu\bar{\mu}} = -1$ , which is compatible with the LEP-II [70] and LHC [71, 72] bounds on lepton couplings.<sup>3</sup>

The  $\Delta F = 2$  constraints on the flavour-violating quark couplings, obtained by a global maximal-likelihood fit to the input parameters given in tables 2 and 3, are shown in the  $\Delta_L^{bq} - \phi_L^{bq}$  plane<sup>4</sup> (the green regions are the 68% and 95% C.L. current allowed regions). In this fit the CKM matrix elements are determined solely by the tree-level constraints, which are not affected by NP. All the hadronic parameters with sizeable uncertainties are treated as nuisance parameters and are marginalised over. The continuous and dashed lines show, in the same plane, the projected sensitivity for NP in  $B_{d,s} \rightarrow \mu^+ \mu^-$  at  $3\sigma$  and  $5\sigma$  as foreseen in 2019 (black) and 2024 (red), using the estimates of table 1. In all these projections we assume no deviations in the  $\Delta F = 2$  observables in order to give the most optimistic prediction for the sensitivity of rare decays. We therefore use the future errors

<sup>3</sup>Flavour-conserving quark couplings of similar size, for the same values of the  $Z'$  mass, are also allowed by the present LHC constraints [73].

<sup>4</sup>With a slight abuse of notation we write here  $\Delta_L^{bq} = \Delta_L^{bq} e^{i\phi_L^{bq}}$ , with  $\Delta_L^{bq}$  real on the right-hand side.



**Figure 1.** Prospects for observing new physics in  $B_s$  (left) and  $B_d$  (right) decays. The green regions show the 68% C.L. and 95% C.L. allowed regions in the  $\Delta F = 2$  fit. The black lines show the  $3\sigma$  (solid) and  $5\sigma$  (dashed) contours for  $\tilde{\mathcal{B}}(B_s \rightarrow \mu^+\mu^-)$  and  $\mathcal{B}(B_d \rightarrow \mu^+\mu^-)$  expected in 2019; the red lines show the same projections for 2024. In both figures  $M_{Z'} = 15$  TeV and  $\Delta_A^{\mu\bar{\mu}} = -3$ .

also for the CKM matrix elements and for the hadronic parameters, assuming SM-like central values. The impact of this choice on the  $\Delta F = 1$  projections is however moderate.

These figures show that already in five years from now it could be possible to probe scales of 15 TeV with rare  $B_s$  decays by observing deviations from the SM predictions at the level of  $3\sigma$ , and reaching a  $5\sigma$  discovery with more data in the following years. On the other hand, for  $B_d$  a  $3\sigma$  effect can be achieved only with the full sensitivity in about ten years from now, for the same value of  $M_{Z'}$ .

The corrections from NP to the Wilson coefficients  $C_9$  and  $C_{10}$ , which weight the semileptonic operators in the effective Hamiltonian relevant for  $b \rightarrow s\mu^+\mu^-$  transitions (see appendix B.5) as used in the recent literature (see e.g. [17, 19, 74–78]) are given as follows [16]

$$\sin^2 \theta_W C_9^{\text{NP}} = -\frac{1}{g_{\text{SM}}^2 M_{Z'}^2} \frac{\Delta_L^{sb} \Delta_V^{\mu\bar{\mu}}}{V_{ts}^* V_{tb}}, \quad (3.8)$$

$$\sin^2 \theta_W C_{10}^{\text{NP}} = -\frac{1}{g_{\text{SM}}^2 M_{Z'}^2} \frac{\Delta_L^{sb} \Delta_A^{\mu\bar{\mu}}}{V_{ts}^* V_{tb}} = -\Delta Y_A(B_s), \quad (3.9)$$

where  $C_9^{\text{NP}}$  involves the leptonic vector coupling of  $Z'$  and  $C_{10}^{\text{NP}}$  the axial-vector one.  $C_9^{\text{NP}}$  plays a crucial role in  $B_d \rightarrow K^*\mu^+\mu^-$  transitions,  $C_{10}^{\text{NP}}$  for  $B_s \rightarrow \mu^+\mu^-$  transitions and both coefficients are relevant for  $B_d \rightarrow K\mu^+\mu^-$ . The  $\text{SU}(2)_L$  relation between the leptonic couplings in (2.2) implies the following important relation [17]

$$-\sin^2 \theta_W C_9^{\text{NP}} = 2\Delta X_L(B_s) + \Delta Y_A(B_s) \quad (3.10)$$

which leads to a triple correlation between  $b \rightarrow s\nu\bar{\nu}$  transitions,  $B_s \rightarrow \mu\bar{\mu}$  and the coefficient  $C_9^{\text{NP}}$  or equivalently  $B_d \rightarrow K^*\mu^+\mu^-$ . Thus even if  $\Delta_L^{\nu\bar{\nu}}$  and  $\Delta_A^{\mu\bar{\mu}}$  are independent of each

other, once they are fixed the values of the coupling  $\Delta_V^{\mu\bar{\mu}}$  and of  $C_9^{\text{NP}}$  are known. We will use these relations in the next section.

Our study of the  $K$  system is eased by the analysis in [31], where an upper bound on the coupling  $\Delta_L^{sd}$  from  $\Delta M_K$  has been derived, assuming conservatively that the NP contribution is at most as large as the short distance SM contribution to  $\Delta M_K$ . Assuming that the NP contribution to  $\Delta M_K$  is at most 30% of its SM value, and rescaling the formula (70) in [31], we find the upper limit

$$|\Delta_L^{sd}| \leq 0.1 \left[ \frac{M_{Z'}}{100 \text{ TeV}} \right], \quad (3.11)$$

which is clearly in the perturbative regime, and is still the case for an  $M_{Z'}$  as large as 2000 TeV. With  $|V_{td}| = 8.5 \times 10^{-3}$  and  $|V_{ts}| = 0.040$  this corresponds to  $|\Delta S(K)| \leq 137$ . Then, again from (3.5), one has, for real  $\Delta_L^{sd}$ ,

$$|\Delta X_L(K)| \leq 0.44 \sqrt{\frac{|\Delta S(K)|}{137}} \left[ \frac{\Delta_L^{\nu\bar{\nu}}}{3.0} \right] \left[ \frac{100 \text{ TeV}}{M_{Z'}} \right]. \quad (3.12)$$

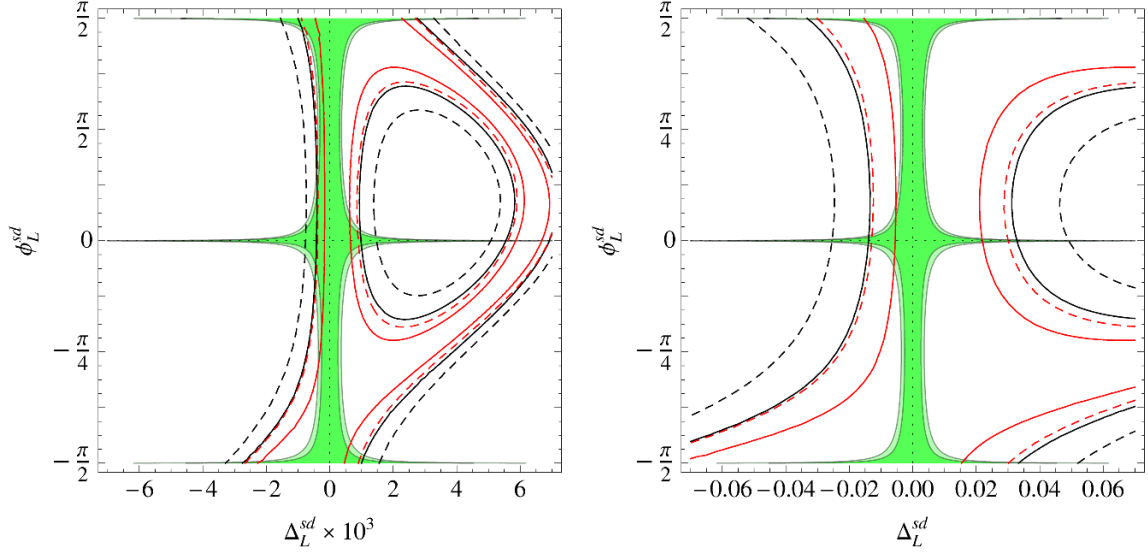
This shift for  $M_{Z'}$  in the ballpark of 100 TeV implies a correction of approximately 50% to the branching ratio for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  but no contribution to  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  since we are assuming  $\Delta_L^{sd}$  to be real. This clearly shows a non-MFV structure of NP because in models with MFV the branching ratio for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is automatically modified when the one for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  is modified. If on the other hand  $\Delta_L^{sd}$  is made complex, significant NP contributions to  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are in general subject to severe constraints from  $\epsilon_K$  and  $\epsilon'/\epsilon$ , unless  $\Delta_L^{sd}$  is purely imaginary, in which case the NP contributions to  $\epsilon_K$  vanish and the effects in  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  are correlated as in MFV. We will perform a more detailed analysis of these two decays and their correlation in section 3.3. Let us discuss here just  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , as this decay will be the first to be measured precisely.

Figure 2 shows the prospects for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , together with the  $\Delta S = 2$  constraints, in the  $\Delta_L^{sd}-\phi_L^{sd}$  plane. We show two different scenarios:

- a beyond-LHC scale of  $M_{Z'} = 50 \text{ TeV}$  with  $\Delta_L^{\nu\bar{\nu}} = 3$ ;
- an LHC scale of  $M_{Z'} = 5 \text{ TeV}$  with  $\Delta_L^{\nu\bar{\nu}} = 1$ .

The conventions and colours are the same as in figure 1. Notice the strong bound from  $\epsilon_K$  for large values of the phase  $\phi_L^{sd}$ , which implies that for NP at high scales with generic CP structure at most a  $3\sigma$  effect can be expected with the precision attainable at the end of the next decade. For real or imaginary couplings, on the contrary, it is evident that scales of 50–100 TeV or even higher may be accessible through  $K$  decays.

The overall message that emerges from the plots in figures 1 and 2 is that through rare meson decays one can resolve energy scales beyond those directly accessible at the LHC: at least in the l.h.s. with suitable values of the  $Z'$  couplings one can still expect deviations from the SM at the level of  $3\text{--}5\sigma$  with the experimental progress of the next few years that are consistent with perturbativity and the meson mixing constraints, for  $M_{Z'}$  in the ranges described above.



**Figure 2.** Prospects for observing new physics in  $K$  decays. The green regions show the 68% C.L. and 95% C.L. allowed regions in the  $\Delta F = 2$  fit. The contours show the  $3\sigma$  and  $5\sigma$  projections for  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  in 2019 and 2024, the colours are as in figure 1. Left:  $M_{Z'} = 5$  TeV and  $\Delta_L^{\nu\bar{\nu}} = 1$ . Right:  $M_{Z'} = 50$  TeV and  $\Delta_L^{\nu\bar{\nu}} = 3$ .

We want to stress once more that the results discussed here correspond to the most optimistic scenarios and to the largest couplings compatible with all considered constraints. Needless to say, in the case of smaller couplings, or in the presence of some approximate flavour symmetry, the scales that may eventually be accessible through rare meson decays are much lower.

### 3.2 Right-handed scenario

If only RH couplings are present the results of the  $\Delta F = 2$  l.h.s. analysis remain unchanged as the relevant hadronic matrix elements — calculated in lattice QCD — are insensitive to the sign of  $\gamma_5$ . Therefore, as far as  $\Delta F = 2$  processes are concerned, it is impossible to state whether in the presence of couplings of only one chirality the deviations from SM expectations are caused by LH or RH currents [16]. In order to make this distinction one has to study  $\Delta F = 1$  processes. In particular in the right-handed scenario (r.h.s.) the relations (3.4) are modified to

$$\Delta Y_A(K) = -\Delta X_R(K) \frac{\Delta_A^{\mu\bar{\mu}}}{\Delta_L^{\nu\bar{\nu}}}, \quad \Delta Y_A(B_q) = -\Delta X_R(B_q) \frac{\Delta_A^{\mu\bar{\mu}}}{\Delta_L^{\nu\bar{\nu}}}, \quad (3.13)$$

where the sign flip plays a crucial role. The functions  $\Delta X_R(M)$  are obtained from  $\Delta X_L(M)$  by replacing the LH quark couplings by the RH ones. We also find for the coefficient of the primed operator  $C'_9$

$$-\sin^2 \theta_W C'_9 = 2\Delta X_R(B_s) + \Delta Y_A(B_s). \quad (3.14)$$

We refer to the appendix A for explicit formulae for all the involved functions.

Therefore the correlations between decays with  $\nu\bar{\nu}$  and  $\mu\bar{\mu}$  in the final state are different in LH and RH scenarios. In particular angular observables in  $B_d \rightarrow K^* \mu^+ \mu^-$  and also the decay  $B_d \rightarrow K \mu^+ \mu^-$  can help in the distinction between l.h.s. and r.h.s., as the presence of RH currents is signalled by the effects of primed operators. In the future the correlation between the decays  $B_d \rightarrow K^* \nu\bar{\nu}$  and  $B_d \rightarrow K \nu\bar{\nu}$  will be able by itself to identify RH currents at work [74, 79–84]. We will show this explicitly in the following sections.

### 3.3 Numerical analysis

We will now perform a numerical study of the  $\Delta F = 1$  effects that can be expected for  $M_{Z'}$  close to its maximal value, and of their correlations. As already indicated by our preceding analysis, the  $\Delta F = 2$  constraints in these scenarios will not allow large  $Z'$  couplings to quarks, but the lepton couplings could be significantly larger than the SM  $Z$  boson couplings, which read<sup>5</sup>

$$\Delta_L^{\nu\bar{\nu}}(Z) = -0.372, \quad \Delta_A^{\mu\bar{\mu}}(Z) = 0.372, \quad \Delta_V^{\mu\bar{\mu}}(Z) = -0.028. \quad (3.15)$$

Working with  $M_{Z'} \geq 15$  TeV we will set

$$\Delta_L^{\nu\bar{\nu}} = \pm 3.0, \quad \Delta_A^{\mu\bar{\mu}} = \mp 3.0, \quad \Delta_V^{\mu\bar{\mu}} = \pm 3.0. \quad (3.16)$$

where the signs are chosen in order to satisfy the  $SU(2)_L$  relation (2.2) in the perturbativity regime. At  $M_{Z'} = 15$  TeV, as well as for the higher masses considered below, these lepton couplings are still consistent with the constraints from LEP-II [70] and the LHC [71, 72].

In our analysis of  $\Delta F = 2$  processes we proceed as follows:

- We set all non-perturbative parameters at their central values. The most important ones are given in table 2. The remaining input can be found in [1]. In order to incorporate effectively the present uncertainties in these parameters we proceed as explained below. See in particular (3.22), (3.24) and (3.25). For future updates see PDG [55], FLAG [44] and HFAG [56].
- For the elements  $|V_{ub}|$  and  $|V_{cb}|$  we use four scenarios corresponding to different determinations from inclusive and exclusive decays with the lower ones corresponding to exclusive determinations. They are given in (3.17)–(3.20) below where we have given the colour coding for these scenarios used in some plots below. The quoted errors are future projections. Arguments have been given recently that NP explanation of the difference between exclusive and inclusive determinations is currently ruled out [85] and must thus be due to underestimated theoretical errors in the form factors and/or the inclusive experimental determination. Finally we use  $\gamma = 68^\circ$ .

The four scenarios for  $|V_{ub}|$  and  $|V_{cb}|$  are given as follows:

$$a) \quad |V_{ub}| = (3.4 \pm 0.1) \times 10^{-3} \quad |V_{cb}| = (39.0 \pm 0.5) \times 10^{-3} \quad (\text{purple}) \quad (3.17)$$

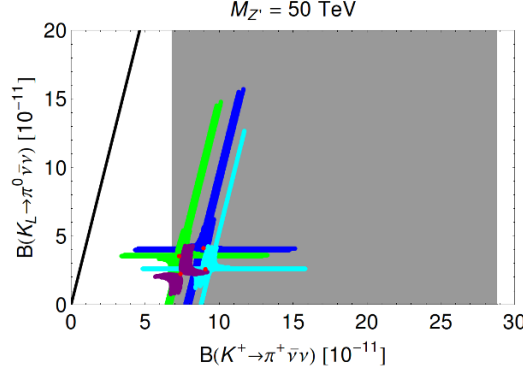
$$b) \quad |V_{ub}| = (3.4 \pm 0.1) \times 10^{-3} \quad |V_{cb}| = (42.0 \pm 0.5) \times 10^{-3} \quad (\text{cyan}) \quad (3.18)$$

$$c) \quad |V_{ub}| = (4.3 \pm 0.1) \times 10^{-3} \quad |V_{cb}| = (39.0 \pm 0.5) \times 10^{-3} \quad (\text{green}) \quad (3.19)$$

$$d) \quad |V_{ub}| = (4.3 \pm 0.1) \times 10^{-3} \quad |V_{cb}| = (42.0 \pm 0.5) \times 10^{-3} \quad (\text{blue}) \quad (3.20)$$

---

<sup>5</sup>For these modified  $Z$  couplings we use the same definition as in (2.4) and (2.5), with  $Z'$  replaced by  $Z$ .



**Figure 3.**  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  versus  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  for  $M_{Z'} = 50$  TeV in the l.h.s.. The colours are as in (3.17)–(3.20). The four red points correspond to the SM central values of the four CKM scenarios, respectively. The black line corresponds to the Grossman-Nir bound [86]. The gray region shows the experimental range of  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$ .

In figure 3 we show the correlation between  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  in the l.h.s. for the four scenarios  $a) - d)$  for  $(|V_{cb}|, |V_{ub}|)$ . To this end we set

$$\Delta_L^{\nu \bar{\nu}} = 3.0, \quad M_{Z'} = 50 \text{ TeV}, \quad (3.21)$$

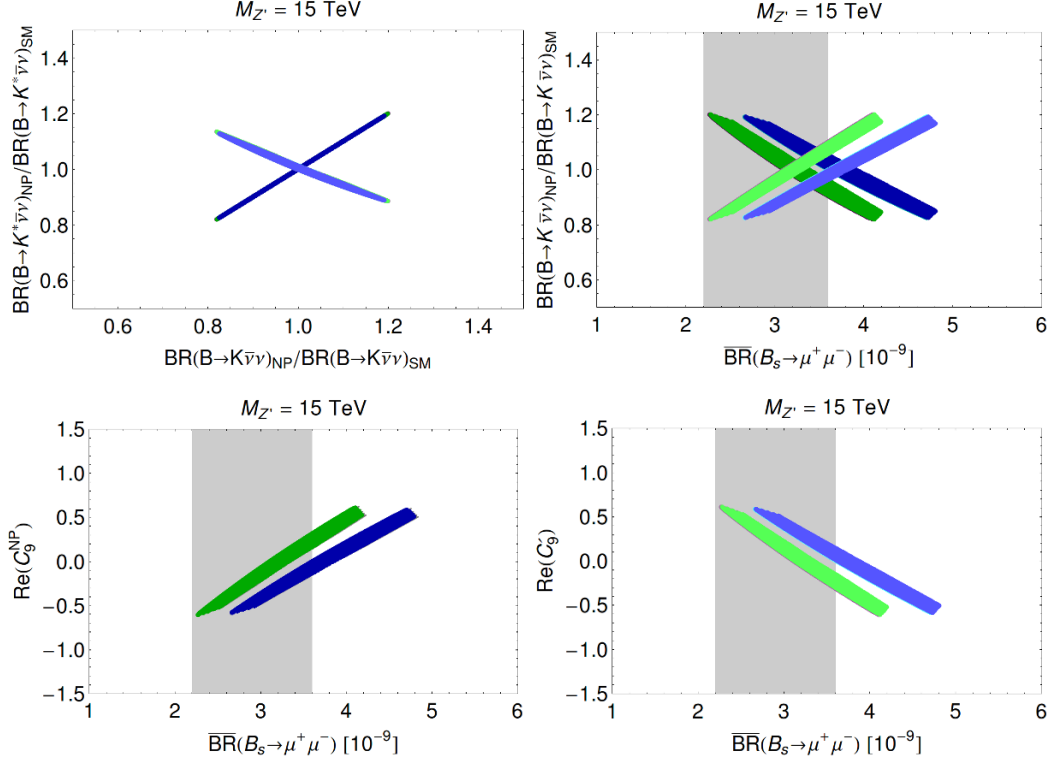
and impose the constraints from  $\Delta M_K$  and  $\varepsilon_K$  by demanding that they are in the ranges

$$0.75 \leq \frac{\Delta M_K}{(\Delta M_K)_{\text{SM}}} \leq 1.25, \quad 2.0 \times 10^{-3} \leq |\varepsilon_K| \leq 2.5 \times 10^{-3}. \quad (3.22)$$

These ranges take into account all other uncertainties beyond CKM parameters such as long distance effects, QCD corrections and the value of  $\gamma$ , which here we keep fixed.

The plot in figure 3 is familiar from other NP scenarios in which the phase of the NP contribution to  $\varepsilon_K$  is twice the one of the NP contribution to  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  [87], as is the case in the scenario considered here.  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  can be strongly enhanced along one of the branches, as a consequence of which  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  will also be enhanced. But  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  can also be enhanced without modifying  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ . The last feature is not possible within the SM and any model with minimal flavour violation, in which these two branching ratios are strongly correlated. The two branches correspond to the regions where the coupling  $\Delta_L^{sd}$  is approximately real or purely imaginary, and the  $\varepsilon_K$  constraint becomes irrelevant, which was already evident in figure 2. For a better analytic understanding of this two branch structure we refer also to [87].

In presenting these results we impose the constraint from  $K_L \rightarrow \mu^+ \mu^-$  in (B.16) which can only have an impact on  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  on the horizontal branch and not on  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ . Because in this scenario the couplings  $\Delta_L^{\nu \bar{\nu}}$  and  $\Delta_A^{\mu \bar{\mu}}$  have opposite signs, in the l.h.s.  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)$  are anti-correlated so that the constraint in (B.16) has no impact on the upper bound on  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ . On the other hand, for the chosen signs of leptonic couplings these two branching ratios are correlated in the RH scenario and the maximal values of  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  on the horizontal branch could in



**Figure 4.** Correlations in the  $B_s$  system for  $M_{Z'} = 15$  TeV in l.h.s. (darker colours) and r.h.s. (lighter colours) with colours as in (3.17)–(3.20). Due to the independence of  $|V_{ub}|$  in this system purple is under green and cyan is under blue. The gray region shows the experimental  $1\sigma$  range  $\overline{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9}$ .

principle be smaller than the ones shown in figure 3 due to the bound in (B.16). However, for the chosen parameters this turns out not to be the case.

As far as the second branch is concerned, as recently analysed in [31] and known from previous literature, the ratio  $\varepsilon'/\varepsilon$  can in principle have a large impact on the largest allowed values of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  on the branch where these branching ratios are correlated. Unfortunately, the present large uncertainties in QCD penguin contributions to  $\varepsilon'/\varepsilon$  do not allow for firm conclusions and we do not show this constraint here.

We observe that large deviations from the SM can be measured even at such high scales. Increasing  $M_{Z'}$  to 100 TeV would reduce NP effects by a factor of two, which could still be measured in the flavour precision era. We conclude therefore that  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decays can probe the Zeptouniverse even if only LH or RH  $Z'$  couplings to quarks are present.

In figure 4 we show the correlations for decays sensitive to  $b \rightarrow s$  transitions. To this end we set in accordance with the signs in (3.16)

$$\Delta_L^{\nu\bar{\nu}} = 3.0, \quad \Delta_A^{\mu\bar{\mu}} = -3.0, \quad \Delta_V^{\mu\bar{\mu}} = 3.0, \quad M_{Z'} = 15 \text{ TeV}. \quad (3.23)$$



$\Delta_L^{\nu\bar{\nu}}$	$\Delta_A^{\mu\bar{\mu}}$	$\Delta_A^{\mu\bar{\mu}}$	(1, 1)	(1, 2)	(2, 1)	(2, 2)
+	+	+	$+(-)$	$+(-)$	−	+
+	−	+	$+(-)$	$-(+)$	+	−
+	−	−	$+(-)$	$-(+)$	−	+

**Table 4.** Correlations (+) and anti-correlations (−) between various observables for different signs of the couplings.  $(n, m)$  denotes the entry in the  $2 \times 2$  matrix in figure 4. For the elements (1, 1) and (1, 2) the signs correspond to l.h.s. (r.h.s.). Flipping simultaneously the signs of all couplings does not change the correlations.

The  $\Delta F = 2$  constraint has been incorporated through the conditions

$$-8^\circ \leq \phi_s \leq 8^\circ, \quad 0.9 \leq C_{B_s} \equiv \frac{\Delta M_s}{\Delta M_s^{\text{SM}}} \leq 1.1 \quad (3.24)$$

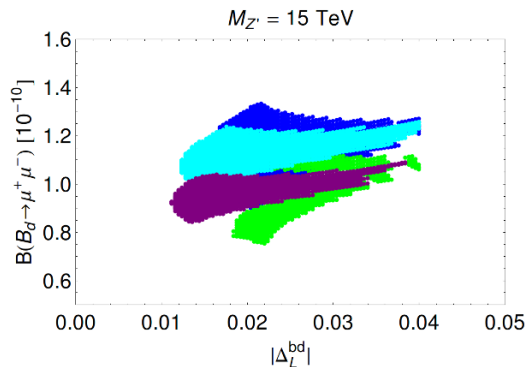
As we have already shown, measurable NP effects are still present at 15 TeV provided the lepton couplings are as large as assumed here, but for larger values of  $M_{Z'}$  the detection of NP would be hard. We consider therefore  $M_{Z'} = 15$  TeV as an approximate upper value in l.h.s. and r.h.s. that can still be probed in the flavour precision era. It will be interesting to monitor the development of the values of  $\phi_s$  and  $C_{B_s}$  in the future. If they will depart significantly from their SM values,  $\phi_s \approx -2^\circ$  and  $C_{B_s} = 1.0$ , NP effects could be observed in rare decays.

In presenting these results we have chosen the leptonic couplings in (3.23), but (3.16) admits a second possibility in which all the couplings are reversed. It is an easy exercise to convince oneself that the correlations presented by us are invariant under this change. On the other hand, for smaller leptonic couplings there are other combinations of the signs of the three leptonic couplings involved that are consistent with perturbativity while satisfying the  $\text{SU}(2)_L$  relation in (2.2). As  $\Delta F = 2$  constraints are independent of leptonic couplings it is not difficult to translate our results into these different possibilities, even if the decrease of leptonic couplings would suppress NP effects. Moreover if the decrease of them was not by a common factor the slopes in our plots would change. This freedom will be important once the experimental data relevant for our plots becomes available. We collect various possibilities in table 4.

Finally in figure 5 we show the branching ratio  $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$  in the l.h.s. as a function of  $|\Delta_L^{bd}|$  for  $M_{Z'} = 15$  TeV, imposing the constraints

$$40^\circ \leq \phi_d \leq 46^\circ, \quad 0.9 \leq C_{B_d} = \frac{\Delta M_d}{\Delta M_d^{\text{SM}}} \leq 1.1. \quad (3.25)$$

As expected, there is a sizeable dependence on the CKM matrix elements. Even if  $B_d^0 - \bar{B}_d^0$  mixing in the SM is strongly suppressed relative to  $B_s^0 - \bar{B}_s^0$  mixing, after the present experimental constraints from  $\Delta F = 2$  observables are imposed the  $B_d$  system allows us to explore approximately the same scales as in the  $B_s$  system. The situation could change when the constraints in (3.24) and (3.25) will be modified in a different manner.



**Figure 5.**  $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$  versus  $|\Delta_L^{bd}|$  for  $M_{Z'} = 15$  TeV in l.h.s., with colours as in (3.17)–(3.20).

## 4 Left-right operators at work

### 4.1 Basic idea

As seen in (3.5), when the constraints from  $\Delta F = 2$  processes are taken into account the  $Z'$  contributions to  $\Delta F = 1$  observables decrease with increasing  $M_{Z'}$ . The reason is simple [16]: a tree-level  $Z'$  contribution to  $\Delta F = 2$  observables depends quadratically on  $\Delta_{L,R}^{ij}/M_{Z'}$ . For any high value of  $M_{Z'}$ , even beyond the reach of the LHC, it is possible to find couplings  $\Delta_{L,R}^{ij}$  which are not only consistent with the existing data but can even remove certain tensions found within the SM. The larger  $M_{Z'}$ , the larger couplings are allowed. Once  $\Delta_{L,R}^{ij}$  are fixed in this manner, they can be used to predict  $Z'$  effects in  $\Delta F = 1$  observables. However here NP contributions to the amplitudes are proportional to  $\Delta_{L,R}^{ij}/M_{Z'}^2$ , and with the couplings proportional to  $M_{Z'}$ , the  $Z'$  contributions to  $\Delta F = 1$  observables decrease with increasing  $M_{Z'}$ .

But this stringent correlation is only present in the l.h.s. and r.h.s. considered until now. If both couplings are present this correlation can be broken, simply because we then have four parameters instead of two in the  $Z'$  couplings to quarks of each meson system. As we will soon see, this will allow us to increase the resolution of short distance scales and allow one to reach Zeptouniverse sensitivities also with the help of  $B_{s,d}$  decays while satisfying their  $\Delta F = 2$  constraints.

### 4.2 L+R scenario

In the presence of both LH and RH couplings of a  $Z'$  gauge boson to SM quarks left-right (LR)  $\Delta F = 2$  operators are generated whose contributions to the mixing amplitudes  $M_{12}^{bq}$  and  $M_{12}^{sd}$  in all three mesonic systems are enhanced through renormalisation group effects relative to left-left (VLL) and right-right (VRR) operators. Moreover in the case of  $M_{12}^{sd}$  additional chiral enhancements of the hadronic matrix elements of LR operators are present. As pointed out in [31] this fact can be used to suppress NP contributions to  $\Delta M_K$  through some fine-tuning between VLL, VRR and LR contributions, thereby allowing for larger contributions to  $K \rightarrow \pi\pi$  amplitudes while satisfying the  $\Delta M_K$  constraint in the limit of small NP phases. Here we generalise this idea to all three systems and NP phases

in  $Z'$  contributions. While the fine-tuning required in the case of  $K \rightarrow \pi\pi$  turned out to be rather large, it will be more modest in the case at hand.<sup>6</sup>

To this end we write the  $Z'$  contributions to the mixing amplitudes as follows [16]:

$$(M_{12}^*)_{Z'}^{sd} = \frac{(\Delta_L^{sd})^2}{2M_{Z'}^2} \langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{sd} z_{sd}, \quad (4.1)$$

and

$$(M_{12}^*)_{Z'}^{bq} = \frac{(\Delta_L^{bq})^2}{2M_{Z'}^2} \langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{bq} z_{bq}, \quad (4.2)$$

where  $z_{sd}$  and  $z_{bq}$  are generally complex. We have

$$z_{sd} = \left[ 1 + \left( \frac{\Delta_R^{sd}}{\Delta_L^{sd}} \right)^2 + 2\kappa_{sd} \frac{\Delta_R^{sd}}{\Delta_L^{sd}} \right], \quad \kappa_{sd} = \frac{\langle \hat{Q}_1^{\text{LR}}(M_{Z'}) \rangle^{sd}}{\langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{sd}} \quad (4.3)$$

with an analogous expressions for  $z_{bq}$ .

Here using the technology of [88, 89] we have expressed  $z_{sd}$  in terms of the renormalisation scheme independent matrix elements

$$\langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{sd} = \langle Q_1^{\text{VLL}}(M_{Z'}) \rangle^{sd} \left( 1 + \frac{11}{3} \frac{\alpha_s(M_{Z'})}{4\pi} \right), \quad (4.4)$$

$$\langle \hat{Q}_1^{\text{LR}}(M_{Z'}) \rangle^{sd} = \langle Q_1^{\text{LR}}(M_{Z'}) \rangle^{sd} \left( 1 - \frac{1}{6} \frac{\alpha_s(M_{Z'})}{4\pi} \right) - \frac{\alpha_s(M_{Z'})}{4\pi} \langle Q_2^{\text{LR}}(M_{Z'}) \rangle^{sd}. \quad (4.5)$$

$\langle Q_1^{\text{VLL}}(M_{Z'}) \rangle^{sd}$  and  $\langle Q_{1,2}^{\text{LR}}(M_{Z'}) \rangle^{sd}$ , which are defined in appendix B.1, are the matrix elements evaluated at  $\mu = M_{Z'}$  in the  $\overline{\text{MS}}$ -NDR scheme, and the presence of  $\mathcal{O}(\alpha_s)$  corrections removes the scheme dependence.  $\alpha_s(M_{Z'})$  is the value of the strong coupling at  $M_{Z'}$ . The corresponding formulae for  $B_q$  mesons are obtained by simply changing  $sd$  to  $bq$  without changing  $\alpha_s$  corrections.

In table 5 we give the central values of the matrix elements in (4.4) and (4.5) for the three meson systems considered and for different values of  $M_{Z'}$ . For the  $K^0 - \bar{K}^0$  system we have used weighted averages of the relevant  $B_i$  parameters obtained in lattice QCD in [90, 91]; for the  $B_{d,s}^0 - \bar{B}_{d,s}^0$  systems we have used the ones in [92]. As the values of the relevant  $B_i$  parameters in these papers have been evaluated at  $\mu = 3 \text{ GeV}$  and  $\mu = 4.29 \text{ GeV}$ , respectively, we have used the formulae in [88] to obtain the values of the matrix elements in question at  $M_{Z'}$ .<sup>7</sup> The renormalisation scheme dependence of the matrix elements is canceled by the one of the Wilson coefficients as mentioned above.

Now, as seen in table 5, both  $\kappa_{sd}$  and  $\kappa_{bq}$  are negative, implying that with the same sign of LH and RH couplings the last term in (4.3) could suppress the contribution of NP to  $\Delta F = 2$  processes. We also note that for  $M_{Z'} \geq 10 \text{ TeV}$  one has  $|\kappa_{sd}| \geq 126$  and  $|\kappa_{bq}| \geq 4.7$  implying that for  $z_{sd}$  and  $z_{bq}$  to be significantly below unity the RH couplings

<sup>6</sup>In order to distinguish this more general scenario from the LRS and ALRS in [16], where the LH and RH couplings were either equal or differed by sign, we denote it simply by L+R.

<sup>7</sup>For simplicity we choose the renormalisation scale to be  $M_{Z'}$ , but any scale of this order would give the same results for the physical quantities up to NNLO QCD corrections that are negligible at these high scales.

$M_{Z'}$	5 TeV	10 TeV	20 TeV	50 TeV	100 TeV	200 TeV
$\langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{sd}$	0.00158	0.00156	0.00153	0.00150	0.00148	0.00146
$\langle \hat{Q}_1^{\text{LR}}(M_{Z'}) \rangle^{sd}$	-0.183	-0.197	-0.211	-0.230	-0.244	-0.259
$\kappa_{sd}(M_{Z'})$	-115.46	-126.51	-137.84	-153.24	-165.20	-177.41
$\langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{bd}$	0.0423	0.0416	0.0409	0.0401	0.0395	0.0390
$\langle \hat{Q}_1^{\text{LR}}(M_{Z'}) \rangle^{bd}$	-0.183	-0.195	-0.206	-0.222	-0.234	-0.246
$\kappa_{bd}(M_{Z'})$	-4.33	-4.68	-5.04	-5.53	-5.92	-6.30
$\langle \hat{Q}_1^{\text{VLL}}(M_{Z'}) \rangle^{bs}$	0.0622	0.0611	0.0601	0.0589	0.0581	0.0573
$\langle \hat{Q}_1^{\text{LR}}(M_{Z'}) \rangle^{bs}$	-0.268	-0.284	-0.301	-0.323	-0.340	-0.357
$\kappa_{bs}(M_{Z'})$	-4.31	-4.66	-5.01	-5.48	-5.85	-6.23

**Table 5.** Central values of the scheme-independent hadronic matrix elements evaluated at different values of  $M_{Z'}$ .  $\langle \hat{Q}_1^{\text{VLL}} \rangle^{ij}$  and  $\langle \hat{Q}_1^{\text{LR}} \rangle^{ij}$  are in units of  $\text{GeV}^3$ .

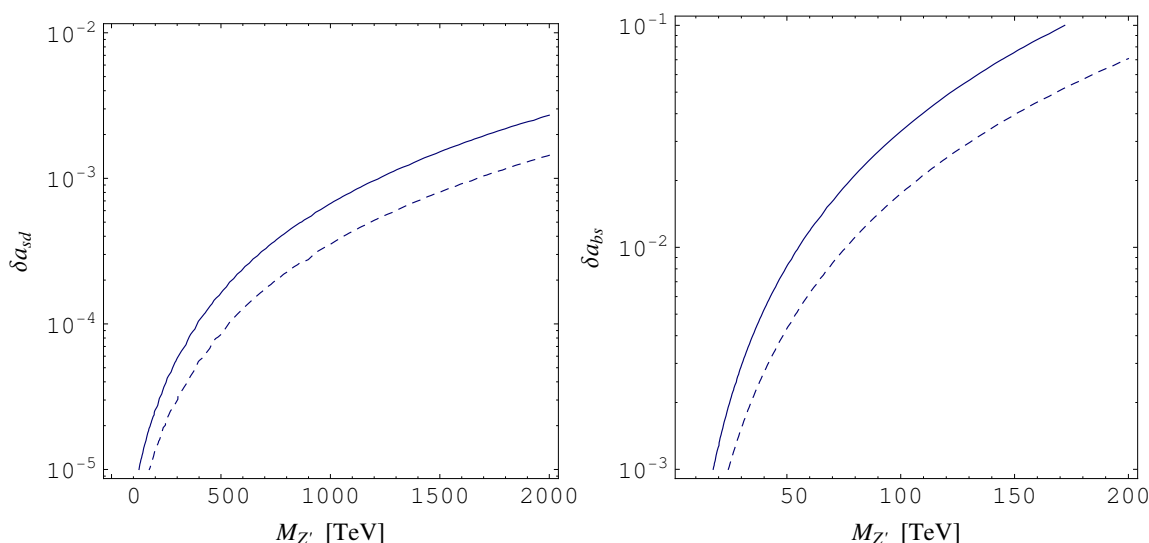
must be much smaller than the LH ones. This in turn implies that the second term in the expression for  $z_{sd}$  in (4.3) can be neglected in first approximation, and we obtain the following hierarchy between LH and RH couplings necessary to suppress NP contributions to  $\Delta F = 2$  observables:

$$\frac{\Delta_R^{sd}}{\Delta_L^{sd}} \simeq -\frac{a_{sd}}{2\kappa_{sd}}, \quad \frac{\Delta_R^{bq}}{\Delta_L^{bq}} \simeq -\frac{a_{bq}}{2\kappa_{bq}}. \quad (4.6)$$

The parameters  $a_{sd}$  and  $a_{bq}$  must be close to unity in order to make the suppression effective. How close they should be to unity depends on present and future results for hadronic and CKM parameters in  $\Delta F = 2$  observables.

Unfortunately the present errors on the hadronic matrix elements are quite large, and do not allow a precise determination of the level of fine-tuning required. An estimate is however possible: in figure 6 we show the deviation of the  $a_{ij}$  from 1,  $\delta a_{ij}$ , allowed by the  $\Delta F = 2$  fit at 68% and 95% C.L. — or, equivalently, the precision up to which the right-handed couplings have to be determined — as a function of  $M_{Z'}$ . In these plots we have fixed the matrix elements in the NP contributions to their central values of table 5, while we included their errors in the SM part. This is justified by our assumption that the SM contribution is the dominant one and gives a good description of data. A shift in the matrix elements  $\kappa_{ij}$  will change the values of  $\Delta_R^{ij}/\Delta_L^{ij}$  that cancel  $z_{ij}$  in (4.3), but the allowed relative deviation from that value, parametrised by  $a_{ij}$ , mainly depends on the error in the SM prediction. In figure 6 for concreteness we have taken maximal phases of  $\pi/4$  for all the couplings and set  $\Delta_L^{ij} = 3$ .

In any case the fact that  $a_{sd}$  and  $a_{bq}$  introduce in each case two new parameters allows us to describe the  $\Delta F = 2$  observables independently of rare decays as opposed to the l.h.s. and r.h.s.. On the other hand, due to the hierarchy of couplings and the absence of LR operators in the rare decays considered by us, rare decays are governed again by LH couplings as in the l.h.s., with the bonus that now the constraint from  $\Delta F = 2$  observables can be ignored. As  $\kappa_{sd} \gg \kappa_{bq}$  the hierarchy of couplings in this scenario must be much larger in the  $K$  system than in the  $B_{s,d}$  systems.



**Figure 6.** Level of fine-tuning in the couplings  $\Delta_R^{sd}$  (left) and  $\Delta_R^{bs}$  (right) required, taking maximal phases and  $\Delta_L^{ij} = 3$ , in order to suppress NP effects in  $\Delta F = 2$  observables in the  $K$  and  $B_s$  systems, respectively, as a function of  $M_{Z'}$ . The dashed and solid lines represent the 68% and 95% C.L. contours.

It is evident from (4.1) and (4.3) that our discussion above remains true if  $L$  and  $R$  are interchanged because the hadronic matrix elements of  $\Delta F = 2$  operators do not depend on the sign of  $\gamma_5$ . In particular the values in table 5 remain unchanged, except that now they apply to the matrix elements of  $Q_1^{\text{VRR}}$  that equal the ones of  $Q_1^{\text{VLL}}$ . In turn  $L$  and  $R$  are interchanged in (4.6) and consequently rare decays are governed by RH couplings in this case. While these two opposite hierarchies cannot be distinguished through  $\Delta F = 2$  observables they can be distinguished through rare decays as we will demonstrate below.

This picture of short distances should be contrasted with the LR and ALR scenarios analysed in [16–19, 93–96], in which the LH and RH couplings were of the same size. In that case the LR operators dominate NP contributions to  $\Delta F = 2$  observables, which implies significantly smaller allowed couplings, and in turn stronger constraints on the  $\Delta F = 1$  observables. Even if also there the signals from LH or RH currents could in principle be observed in rare  $K$  and  $B_{sd}$  decays, their effects will only be measurable for scales below 10 TeV.

The main message of this section is the following one: by appropriately choosing the hierarchy between LH and RH flavour violating  $Z'$  couplings to quarks one can eliminate to a large extent the constraints from  $\Delta F = 2$  transitions even in the presence of large CP-violating phases, and in this manner increase the resolution of short distance scales, which now would be probed solely by rare  $K$  and  $B_{sd}$  decays. While in the  $B_{d,s}$  systems this can be done at the price of a mild fine-tuning, and allows one to reach the Zeptouniverse, in the  $K$  system it requires a fine-tuning of the couplings at the level of 1%–1‰ because of the strong  $\varepsilon_K$  constraint (see figure 6). Notice however that  $K$  decays already allowed us to reach 100 TeV in the l.h.s. without the need of right-handed couplings.

The implications of this are rather profound. Even if in the future SM would agree perfectly with all  $\Delta F = 2$  observables, this would not necessarily imply that no NP effects can be seen in rare decays, even if the  $Z'$  is very heavy. The maximal value of the  $Z'$  mass,  $M_{Z'}^{\max}$ , for which measurable effects in rare decays could in principle still be found, and perturbativity of couplings is respected, is again rather different in different systems, and depends on the assumed perturbativity upper bounds on  $Z'$  couplings and on the sensitivity of future experiments.

In appendix B we give expressions for the rare decay branching ratio observables  $\mathcal{B}$  given in table 1, which depend on the functions  $X_{L,R}$  and  $Y_{L,R}$  listed in appendix A. Combining these formulae gives the following relation for a non-zero  $\Delta X_L(M)$  (as defined in (3.3))

$$M_{Z'}^{\max} = K(M) \sqrt{\left| \frac{\Delta_L^{\nu\bar{\nu}}}{3.0} \right|} \sqrt{\left| \frac{\Delta_L^{ij}}{3.0} \right|} \sqrt{\left| \frac{10\%}{\delta_{\text{exp}}(M)} \right|}, \quad (4.7)$$

where  $ij = sd, db, sb$  for  $M = K, B_d, B_s$ , respectively, and  $\delta_{\text{exp}}(M) \equiv \delta\mathcal{B}/\mathcal{B}$  is the experimental sensitivity that can be reached in  $M$  decays, as listed in table 1. For the present CKM parameters the factors  $K(M)$  are as follows:

$$K(K) \approx 1400 \text{ TeV}, \quad K(B_d) \approx 280 \text{ TeV}, \quad K(B_s) \approx 140 \text{ TeV}. \quad (4.8)$$

One has similar formulae for  $Y_A(M)$ , but as  $Y^{\text{SM}} \approx 0.65 X^{\text{SM}}$  one can reach slightly higher values of  $M_{Z'}$  for the same experimental sensitivity. We note that this time there is a difference between the  $B_d$  and  $B_s$  system, which was not the case in section 3. We also note that, although these maximal values depend on the assumed maximal values of the  $Z'$  couplings to SM fermions and the assumed sensitivity to NP, this is not a strong dependence due to the square roots involved. Using the projections for 2024 in table 1, we get

$$M_{Z'}^{\max}(K) \approx 2000 \text{ TeV}, \quad M_{Z'}^{\max}(B_s) \approx M_{Z'}^{\max}(B_d) \approx 160 \text{ TeV}, \quad (4.9)$$

so that  $M_{Z'}^{\max}$  in  $B_s$  and  $B_d$  systems are comparable in spite of the difference in the factors  $K(M)$  in (4.8).

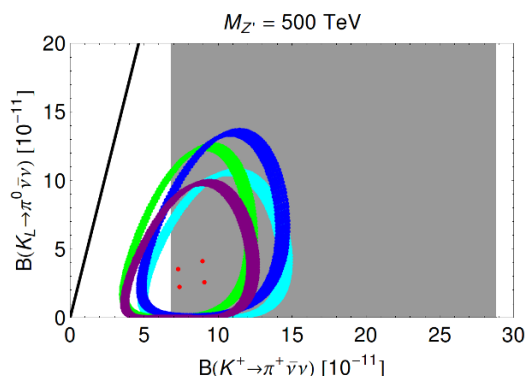
### 4.3 Numerical analysis

Our analysis of this scenario follows the one of section 3.3 except that now we may ignore the  $\Delta F = 2$  constraints and increase all left-handed quark couplings (in the case of the dominance of left-handed currents) to

$$\Delta_L^{sd} = 3.0 e^{i\phi_L^{sd}}, \quad \Delta_L^{bd} = 3.0 e^{i\phi_L^{bd}}, \quad \Delta_L^{bs} = 3.0 e^{i\phi_L^{bs}} \quad (4.10)$$

with arbitrary phases  $\phi_L^{ij}$ . For the lepton couplings we use the values given in (3.16).

In figure 7 we show the correlation between  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  for the four scenarios  $a) - d)$  for  $(|V_{cb}|, |V_{ub}|)$  and  $M_{Z'} = 500 \text{ TeV}$ . The pattern of correlations in figure 7 is very different from the one in figure 3 as now the phase of the NP contribution to  $\varepsilon_K$  is generally not twice the one of the NP contribution to  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ . Therefore, as already discussed in general terms in [87] the two branch structure seen in



**Figure 7.**  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  versus  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  for  $M_{Z'} = 500$  TeV in L+R scenario. The colours are as in (3.17)–(3.20). The four red points correspond to the SM central values of the four CKM scenarios, respectively. The black line corresponds to the Grossman-Nir bound [86]. The gray region shows the experimental range of  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$ .

figure 3 is absent here. In particular, it is possible to obtain values for  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  that are outside the two branches seen in figure 3 and that differ from the SM predictions. This feature could allow us to distinguish these two scenarios. It should also be added that without  $\Delta F = 2$  constraints NP effects at the level of the amplitude decrease quadratically with increasing  $M_{Z'}$  so that for  $M_{Z'} = 1000$  TeV NP would contribute only at the 15% level. While such small effects are impossible to detect in other decays considered by us, the exceptional theoretical cleanness of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  could in principle allow to study such effect one day. On the other hand for  $M_{Z'} = 200$  TeV the enhancements of both branching ratios could be much larger than shown in figure 7. This would require higher fine-tuning in the  $\Delta F = 2$  sector as seen in figure 6.

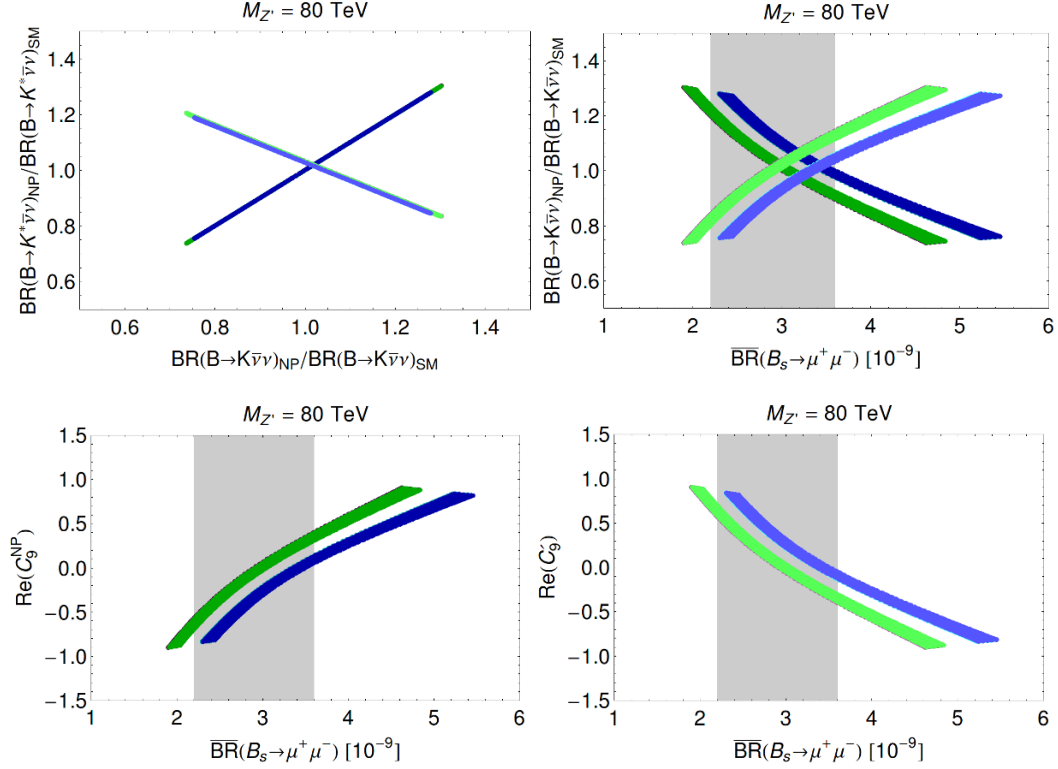
As we fixed the absolute values of the couplings in this example, the different values of branching ratios on the circles correspond to different values of the phase  $\phi_L^{sd}$ , when it is varied from 0 to  $2\pi$ . Measuring these two branching ratios would determine this phase uniquely. Most importantly, we observe that even at such high scales NP effects are sufficiently large to be measured in the future.

In figure 8 we show various correlations sensitive to the  $\Delta_{L,R}^{bs}$  couplings in L+R scenario for  $M_{Z'} = 80$  TeV. The choice of lepton couplings is as in (3.23).

We observe the following features:

- The correlations have this time very similar structure to the one found in figure 4 for  $M_{Z'} = 15$  TeV but due to larger quark couplings and the absence of  $\Delta F = 2$  constraints NP effects can be sizeable even at  $M_{Z'} = 80$  TeV.
- As expected, a clear distinction between LH and RH couplings can be made provided NP effects in  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  will be sufficiently large in order to allow measurable NP effects in other four observables shown in the figure 8.

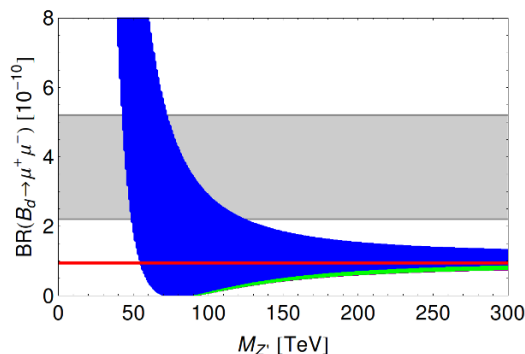




**Figure 8.** Correlations in the  $B_s$  system for  $M_{Z'} = 80$  TeV in L+R scenario (colours as in (3.17)–(3.20) but with much overlap, due to the very weak dependence on  $|V_{ub}|$ , i.e. purple is under green and cyan is under blue). Darker colours correspond to the scenario where LH couplings dominate over RH and vice versa for lighter colours. The gray region shows the experimental  $1\sigma$  range in  $\overline{B}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9}$ .

Due to the similarity of the plots in figures 4 and 8 the question arises how we could distinguish these two scales through future measurements. While some ideas for this distinction will be developed in section 7, here we just want to make the following observation. Once the values of  $S_{\psi\phi}$  and  $C_{B_s}$  will be much more precisely known than assumed in (3.24), the range of allowed values for the observables in figure 4 will be significantly decreased, possibly ruling out this scenario through rare decay measurements. On the other hand this progress in the determination of  $\Delta F = 2$  observables will have no impact on the plots in figure 8 allowing the theory to pass these constraints.

Finally, in figure 9 we show  $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$  as a function of  $M_{Z'}$  together with the SM prediction and the experimental range. We observe that even for  $M_{Z'} = 200$  TeV there are visible departures from the SM prediction. For  $M_{Z'} = 50$  TeV even the present  $1\sigma$  experimental range can be reached. This plot shows that for even smaller values of  $M_{Z'}$  interesting results with smaller couplings can be obtained.



**Figure 9.**  $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$  versus  $M_{Z'}$  in the L+R scenario. The red line corresponds to SM central value and grey area is the experimental region:  $(3.6^{+1.6}_{-1.4}) \times 10^{-10}$ .

## 5 The case of a neutral scalar or pseudoscalar

### 5.1 Preliminaries

Tree-level neutral scalar and pseudo-scalar exchanges<sup>8</sup> can give large contributions to  $\Delta F = 2$  and  $\Delta F = 1$  processes. Prominent examples are supersymmetric theories at large  $\tan \beta$ , two-Higgs doublet models (2HDMs) and left-right symmetric models. In the case of  $\Delta F = 2$  transitions new scalar operators are generated and in the presence of  $\mathcal{O}(1)$  flavour-violating couplings one can be sensitive to scales as high as  $10^4$  TeV, or even more [3–5]. The question then arises which distance scales can be probed by  $\Delta F = 1$  processes mediated by tree-level scalar exchanges. In order to answer this question in explicit terms we will concentrate here on the decays  $B_{s,d} \rightarrow \mu^+ \mu^-$ , which, as we will momentarily show, allow to reach the Zeptouniverse without any fine-tuning in the presence of new heavy scalars with large couplings to quarks and leptons. As we have seen in section 3 this was not possible in the case of a heavy  $Z'$ . We have checked that other decays analysed in the previous sections cannot compete with  $B_{s,d} \rightarrow \mu^+ \mu^-$  in probing very short distance scales in the presence of neutral heavy scalars with flavour-violating couplings. In fact, as we will see,  $B_{s,d} \rightarrow \mu^+ \mu^-$  play a prominent role in testing very short distance scales in this case, as  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  play in  $Z'$  NP scenarios.

A very detailed analysis of generic scalar tree-level contributions to  $\Delta F = 2$  and  $\Delta F = 1$  processes has been presented in [94, 95]. In particular in [95] general formulae for various observables have been presented. We will not repeat these formulae here but we will use them to derive a number of expressions that will allow us a direct comparison of this NP scenario with the  $Z'$  one.

Our goal then is to find out what is the highest energy scale which can be probed by  $B_{s,d} \rightarrow \mu^+ \mu^-$  when the dominant NP contributions are tree-level scalar exchanges subject to present  $\Delta F = 2$  constraints and perturbativity. We will first present general expressions and subsequently we will discuss in turn the cases analogous to the  $Z'$  scenarios of sections 3 and 4.

<sup>8</sup>In what follows, unless specified, we will use the name *scalar* for both scalars and pseudo-scalars.

## 5.2 General formulae

Denoting by  $H$  a neutral scalar with mass  $M_H$  the mixing amplitudes are given as follows ( $q = s, d$ )

$$(M_{12}^*)_{H}^{bq} = - \left[ \frac{(\Delta_L^{bq}(H))^2}{2M_H^2} + \frac{(\Delta_R^{bq}(H))^2}{2M_H^2} \right] \langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bq} - \frac{\Delta_L^{bq}(H)\Delta_R^{bq}(H)}{M_H^2} \langle \hat{Q}_2^{\text{LR}}(M_H) \rangle^{bq}. \quad (5.1)$$

Here  $\Delta_{L,R}^{bq}(H)$  are the left-handed and right-handed scalar couplings and the renormalisation scheme independent matrix elements are given as follows [89]

$$\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bq} = \langle Q_1^{\text{SLL}}(M_H) \rangle^{bq} \left( 1 + \frac{9}{2} \frac{\alpha_s(M_H)}{4\pi} \right) + \frac{1}{8} \frac{\alpha_s(M_H)}{4\pi} \langle Q_2^{\text{SLL}}(M_H) \rangle^{bq}, \quad (5.2)$$

$$\langle \hat{Q}_2^{\text{LR}}(M_H) \rangle^{bq} = \langle Q_2^{\text{LR}}(M_H) \rangle^{bq} \left( 1 - \frac{\alpha_s(M_H)}{4\pi} \right) - \frac{3}{2} \frac{\alpha_s(M_H)}{4\pi} \langle Q_1^{\text{LR}}(M_H) \rangle^{bq}. \quad (5.3)$$

The operators  $Q_{1,2}^{\text{SLL}}$  are defined in appendix B.1. The operators  $Q_{1,2}^{\text{LR}}$  were already present in the case of  $Z'$  but now, as seen from (5.3), the operator  $Q_2^{\text{LR}}$  plays the dominant role. In writing (5.1) we have used the fact that the matrix elements of the RH scalar operators  $Q_{1,2}^{\text{SRR}}$  equal those of  $Q_{1,2}^{\text{SLL}}$  operators. The Wilson coefficients of  $Q_{1,2}^{\text{SRR}}$  are represented in (5.1) by the term involving  $(\Delta_R^{bq}(H))^2$ .

In analogy to (4.2) we can rewrite (5.1)

$$(M_{12}^*)_{H}^{bq} = - \frac{(\Delta_L^{bq}(H))^2}{2M_H^2} \langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bq} \tilde{z}_{bq}(M_H), \quad (5.4)$$

where  $\tilde{z}_{bq}(M_H)$  is generally complex, and is given by

$$\tilde{z}_{bq}(M_H) = \left[ 1 + \left( \frac{\Delta_R^{bq}(H)}{\Delta_L^{sd}(H)} \right)^2 + 2\tilde{\kappa}_{bq}(M_H) \frac{\Delta_R^{bq}(H)}{\Delta_L^{bq}(H)} \right], \quad (5.5)$$

$$\tilde{\kappa}_{bq}(M_H) = \frac{\langle \hat{Q}_2^{\text{LR}}(M_H) \rangle^{sd}}{\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{sd}}. \quad (5.6)$$

In table 6 we give the central values of the renormalization scheme independent matrix elements of (5.2) and (5.3) for the three meson systems and for different values of  $M_H$ , using the lattice results of [90–92] as in table 5. For simplicity we set the renormalisation scale to  $M_H$ , but any scale of this order would give the same results for the physical quantities up to NNLO QCD corrections that are negligible at these high scales. We also give the values of  $\tilde{\kappa}_{bq}(M_H)$  and of  $m_b(M_H)$  that we will need below. The results for the K system are given here only for completeness but we will not study rare  $K$  decays in this section as they are not as powerful as  $B_{s,d} \rightarrow \mu^- \mu^-$  in probing short distance scales in the scalar NP scenarios.

We have summarised the formulae for the branching ratio observables of  $B_{s,d} \rightarrow \mu^+ \mu^-$  decays in appendix B.6. In the case of tree-level scalar and pseudo-scalar exchanges, the

$M_H$	5 TeV	10 TeV	20 TeV	50 TeV	100 TeV	200 TeV	500 TeV	1000 TeV
$\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{sd}$	-0.089	-0.093	-0.096	-0.101	-0.105	-0.108	-0.113	-0.116
$\langle \hat{Q}_2^{\text{LR}}(M_H) \rangle^{sd}$	0.291	0.312	0.334	0.362	0.384	0.405	0.434	0.456
$\tilde{\kappa}_{sd}(M_H)$	-3.27	-3.37	-3.46	-3.58	-3.66	-3.75	-3.86	-3.94
$\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bd}$	-0.095	-0.099	-0.103	-0.108	-0.112	-0.116	-0.120	-0.124
$\langle \hat{Q}_2^{\text{LR}}(M_H) \rangle^{bd}$	0.245	0.262	0.280	0.304	0.322	0.340	0.365	0.383
$\tilde{\kappa}_{bd}(M_H)$	-2.57	-2.64	-2.72	-2.81	-2.88	-2.95	-3.03	-3.09
$\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bs}$	-0.140	-0.146	-0.152	-0.159	-0.164	-0.170	-0.177	-0.182
$\langle \hat{Q}_2^{\text{LR}}(M_H) \rangle^{bs}$	0.348	0.373	0.399	0.432	0.458	0.484	0.519	0.545
$\tilde{\kappa}_{bs}(M_H)$	-2.48	-2.56	-2.63	-2.72	-2.79	-2.85	-2.93	-2.99
$m_b(M_H)[\text{GeV}]$	2.27	2.19	2.12	2.03	1.97	1.92	1.85	1.81

**Table 6.** Central values of the scheme-independent hadronic matrix elements evaluated at different values of  $M_H$ .  $\langle \hat{Q}_1^{\text{SLL}} \rangle^{ij}$  and  $\langle \hat{Q}_2^{\text{LR}} \rangle^{ij}$  are in units of  $\text{GeV}^3$ .

Wilson coefficients of the corresponding effective Hamiltonian (see e.g. [95]), which vanish in the SM, are given as follows

$$m_b(\mu_H) \sin^2 \theta_W C_S^{(\prime)} = \frac{1}{g_{\text{SM}}^2} \frac{1}{M_H^2} \frac{\Delta_{R(L)}^{bq}(H) \Delta_S^{\mu\bar{\mu}}(H)}{V_{tq}^* V_{tb}}, \quad (5.7)$$

$$m_b(\mu_H) \sin^2 \theta_W C_P^{(\prime)} = \frac{1}{g_{\text{SM}}^2} \frac{1}{M_H^2} \frac{\Delta_{R(L)}^{bq}(H) \Delta_P^{\mu\bar{\mu}}(H)}{V_{tq}^* V_{tb}}, \quad (5.8)$$

where  $\Delta_{S,P}^{\mu\bar{\mu}}(H)$  are given by

$$\begin{aligned} \Delta_S^{\mu\bar{\mu}}(H) &= \Delta_R^{\mu\bar{\mu}}(H) + \Delta_L^{\mu\bar{\mu}}(H), \\ \Delta_P^{\mu\bar{\mu}}(H) &= \Delta_R^{\mu\bar{\mu}}(H) - \Delta_L^{\mu\bar{\mu}}(H), \end{aligned} \quad (5.9)$$

such that the corresponding Lagrangian reads [95]

$$\mathcal{L} = \frac{1}{2} \bar{\mu} [\Delta_S^{\mu\bar{\mu}}(H) + \gamma_5 \Delta_P^{\mu\bar{\mu}}(H)] \mu H. \quad (5.10)$$

$\Delta_S^{\mu\bar{\mu}}$  is real and  $\Delta_P^{\mu\bar{\mu}}$  purely imaginary as required by the hermiticity of the Hamiltonian. See [95] for properties of these couplings. It should be noted that  $C_S$  and  $C_P$  involve the scalar right-handed quark couplings, whereas  $C'_S$  and  $C'_P$  the left-handed ones.

An important feature to be stressed here is that for the same values of the couplings  $\Delta_S^{\mu\bar{\mu}}(H)$  and  $\Delta_P^{\mu\bar{\mu}}(H)$  the pseudoscalar contributions play a more important role because they interfere with the SM contributions (see (B.23)). Therefore, in order to find the maximal values of  $M_H$  that can be tested by  $B_{s,d} \rightarrow \mu^+ \mu^-$ , it is in principle sufficient to consider only the pseudoscalar contributions  $P$ . But for completeness we will also show the results for the scalar case.

### 5.3 Left-handed and right-handed scalar scenarios

These two scenarios correspond to the ones considered in section 3 and involve respectively either only LH scalar currents (SLL scenario) or RH ones (SRR scenario). In these simple cases it is straightforward to derive the correlations between pseudoscalar contributions to  $\Delta F = 2$  observables and the values of the Wilson coefficients  $C_P$  and  $C'_P$ . One finds

$$m_b(\mu_H) \sin^2 \theta_W \frac{C_P^{(\prime)}(B_q)}{\sqrt{[\Delta S(B_q)]_{RR(LL)}^*}} = \frac{\Delta_P^{\mu\bar{\mu}}(H)}{2 M_H g_{SM}} \sqrt{\frac{\langle Q_1^{\text{VLL}}(m_t) \rangle^{bq}}{-\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bq}}} \\ = 0.0015 \Delta_P^{\mu\bar{\mu}}(H) \left[ \frac{500 \text{ TeV}}{M_H} \right], \quad (5.11)$$

where  $[\Delta S(B_q)]_{LL}$  and  $[\Delta S(B_q)]_{RR}$  are the shifts in the SM one-loop  $\Delta F = 2$  function  $S^{\text{SM}}$  caused by the pseudoscalar tree-level exchanges in SLL and SRR scenarios respectively. The matrix elements  $\langle \hat{Q}_1^{\text{SLL}}(M_H) \rangle^{bq}$  are given for various values of  $M_H$  in table 6, while the  $\langle Q_1^{\text{VLL}}(m_t) \rangle^{bq}$  evaluate to  $0.046 \text{ GeV}^3$  and  $0.067 \text{ GeV}^3$  for  $q = d$  and  $q = s$ , respectively.

In order to find the maximal values of  $M_H$  that can be tested by future measurements we assume

$$\Delta_P^{\mu\bar{\mu}}(H) = 3.0 i, \quad |[\Delta S(B_q)]_{LL}| \leq 0.36. \quad (5.12)$$

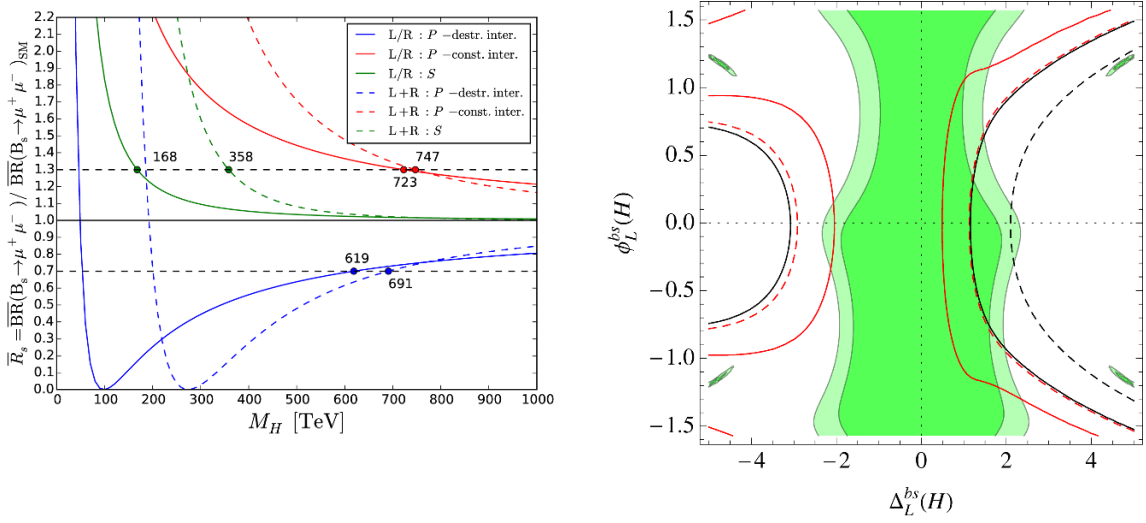
Then by using the formulae listed above we can calculate the ratio  $\bar{R}_q$  of  $\bar{\mathcal{B}}(B_q \rightarrow \mu^+ \mu^-)$  to its SM expectation, given in (B.22), as a function of  $M_H$ . From table 1 we see that in 2024 a deviation of  $3\sigma$  from the SM estimate of  $\bar{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)$  will correspond to a 30% deviation in  $\bar{R}_s$  from one. In the left panel of figure 10 we show the dependence of  $\bar{R}_s$  on  $M_H$  for the case of pseudo-scalar and scalar exchanges. We observe that measurable effects of pseudo-scalar exchanges can be obtained at  $M_H$  as high as 600–700 TeV for the large couplings considered, which is also dependent on constructive or destructive interference with the SM. Because scalars do not interfere with the SM contributions, they only just approach the Zeptouniverse scale of 200 TeV.

In the right panel of figure 10 we show the result of a fit of all the  $\Delta F = 2$  constraints for an arbitrary phase of the  $\Delta_L^{bs}(H)$  coupling — i.e. allowing for CP violation in the scalar sector — together with the projections for  $\bar{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)$  in 2019 and 2024, in the plane<sup>9</sup>  $\Delta_L^{bs}(H) - \phi_L^{bs}(H)$ . The notation is the same as in figure 1, with the green regions being allowed by the  $\Delta F = 2$  fit at 68% and 95% C.L., and the continuous and dashed lines indicating  $3\sigma$  and  $5\sigma$  effects in  $B_s \rightarrow \mu^+ \mu^-$ , respectively. We fixed  $M_H = 500 \text{ TeV}$  and  $|\Delta_P^{\mu\mu}| = 3$ . The effects are maximal for real, positive values of the coupling, where there is maximal constructive interference with the SM contribution.

Lower precision is expected for  $\bar{\mathcal{B}}(B_d \rightarrow \mu^+ \mu^-)$  in the LHC era, with a  $3\sigma$  effect corresponding to a 60% deviation in  $\bar{R}_d$  by 2030. Therefore with equivalent constraints on  $B_d$  mixing as given in (5.12) the scales that can be probed in the SLL or SRR scenarios are lower, yet still within the Zeptouniverse.

The maximal effects given here are of course lower for smaller values of the scalar lepton couplings  $\Delta_{S,P}^{\mu\mu}$ , as expected in most motivated concrete models.

<sup>9</sup>Writing the  $\Delta_L^{bq}(H)$  coupling as  $i\Delta_L^{bq}(H)e^{i\phi_L^{bq}(H)}$ .



**Figure 10.** Left: dependence of  $\bar{R}_s$  on the heavy scalar mass  $M_H$ , showing the pure LH (or RH) scenario and the combined L+R scenario (see text for details). Right: analogous situation to figure 1, but for a heavy pseudo-scalar with  $M_H = 500$  TeV and  $|\Delta_P^{\mu\mu}(M_H)| = 3$  in the l.h.s. .

#### 5.4 L+R scalar scenario

In the presence of both LH and RH couplings the  $\Delta F = 2$  constraints can be loosened so that higher values of  $M_H$  can be probed. Let us again set  $|\Delta_L^{bq}| = 3$ , consistent with perturbativity bounds. In order for NP effects in  $B_q$  mixing to be negligible, we require  $\Delta_R^{bq}$  to be such that  $\tilde{z}_{bq}(M_H)$  given in (5.5) approximately vanish. This happens when

$$\Delta_R^{bq} \approx - \left( \tilde{\kappa}_{bq}(M_H) \pm \sqrt{\tilde{\kappa}_{bq}(M_H)^2 - 1} \right) \Delta_L^{bq} \sim \frac{1}{5} \Delta_L^{bq}, \quad (5.13)$$

where in the last expression we have kept only the “+” solution in order to be consistent with perturbativity. As we already discussed in the previous sections, interchanging  $L$  and  $R$  in (5.13) and setting  $|\Delta_R^{bq}| = 3$  is also a solution. In the left panel of figure 10 we show the dependence of  $\bar{R}_s$  on  $M_H$  for the case of pseudo-scalar and scalar exchanges also for this scenario. We observe that, for measurable effects in  $\bar{R}_s$  greater than 30%, scales of 700–750 TeV can be probed, which are only slightly higher as compared to the pure SLL or SRR cases. This is easily understood in terms of the fact that flavour-violating couplings of order 2–3, close to their perturbativity bound, were allowed by the  $\Delta F = 2$  constraints already in the pure SLL and SRR cases (see the right panel of figure 10), giving also there large effects in  $B_{s,d}$  decays.

In contrast, for NP effects that give a  $\bar{R}_d$  greater than 60%, which could be observable at  $3\sigma$  in 2030, the additional smallness of the  $\bar{\mathcal{B}}(B_d \rightarrow \mu^+ \mu^-)$  SM estimate (due to  $|V_{td}| \ll |V_{ts}|$ ) allows scales up to 1200 TeV to be probed for the large couplings we consider.

## 6 Other New Physics scenarios

### 6.1 Preliminaries

We would like now to address the question whether our findings can be generalised to other NP scenarios while keeping in mind that we would like to obtain the highest possible resolution of short distance scales with the help of  $\Delta F = 1$  processes and staying consistent with the constraints from  $\Delta F = 2$  processes and perturbativity. After all our NP scenarios up till now have been very simple: one heavy gauge boson or (pseudo-)scalar contributing to both  $\Delta F = 1$  and  $\Delta F = 2$  transitions at tree-level. In general one could have several new particles and, moreover, there is the possibility of a GIM mechanism at work protecting against FCNCs at tree-level. Before discussing various possibilities let us make a few general observations:

- If a gauge boson or scalar (pseudoscalar) contributes at tree-level to  $\Delta F = 1$  transitions it will necessarily contribute also to  $\Delta F = 2$  transitions.
- On the other hand a gauge boson or a scalar (pseudoscalar) can contribute to  $\Delta F = 2$  transitions at tree-level without having any impact on  $\Delta F = 1$  transitions. This is the case, for instance, for a heavy gluon  $G'$ , which, carrying colour, does not couple to leptons, or for a leptophobic  $Z'$ . In the case of a scalar (pseudoscalar) this could be realised if the coupling of these bosons to leptons is suppressed through small lepton masses, which is the case if these bosons take part in electroweak symmetry breaking.

We will now briefly discuss two large classes of NP models, reaching the following conclusions:

- In order to achieve a high resolution of short-distance scales in the presence of tree-level FCNCs that satisfy  $\Delta F = 2$  constraints, one generally has to break the correlation between  $\Delta F = 1$  and  $\Delta F = 2$  transitions. In the case of a single  $Z'$  or (pseudo-)scalar this can be done via the L+R scenario, or by the introduction of multiple such NP particles.<sup>10</sup>
- If the GIM mechanism is at work and there are no tree-level FCNCs the pattern of correlations between  $\Delta F = 1$  and  $\Delta F = 2$  transitions could be different than in the case of tree-level FCNCs. Yet, as we will show, in this case the energy scales which can be explored by rare  $K$  and  $B_{s,d}$  decays are significantly lower than the ones found by us in the previous sections.

### 6.2 The case of two gauge bosons

Let us assume that there are two gauge bosons  $Z'_1$  and  $Z'_2$  but only  $Z'_1$  couples to leptons i.e.  $Z'_2$  could be colourless or an octet of  $SU(3)_c$ . In such a model it is possible to reach very high scales with only LH or RH couplings to quarks. Indeed, let us assume that these two bosons have only LH flavour violating couplings. Only  $Z'_1$  is relevant for  $\Delta F = 1$

<sup>10</sup>In special cases such as the decays  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  in  $Z'$  scenarios, and  $B_{s,d} \rightarrow \mu^+ \mu^-$  in the scalar case, the Zeptouniverse can be reached even in the presence of  $\Delta F = 2$  constraints.



transitions and if  $Z'_2$  were absent we would have the LH scenario of section 3, which does not allow measurable effects in  $B_{s,d}$  decays above 20 TeV due to  $\Delta F = 2$  constraints.

On the contrary, with two gauge bosons we can suppress NP contributions to  $\Delta F = 2$  transitions by choosing their couplings and masses such that their contributions to  $\Delta M_{s,d}$  approximately cancel. This is clearly a tuned scenario. Assuming that the masses of these bosons are of the same order so that we can ignore the differences in RG QCD effects, a straightforward calculation allows us to derive the relation

$$\left[ \frac{\Delta_L^{ij}(Z'_1)}{\Delta_L^{ij}(Z'_2)} \right]^2 = -\frac{1}{N_c} \left[ \frac{M_{Z'_1}}{M_{Z'_2}} \right]^2 \quad (6.1)$$

which should be approximately satisfied. Here  $N_c$  is equal to 3 or 1 for  $Z'_2$  with or without colour, respectively. This in turn implies

$$\Delta_L^{ij}(Z'_2) = i\sqrt{N_c} \Delta_L^{ij}(Z'_1) \left[ \frac{M_{Z'_2}}{M_{Z'_1}} \right] \quad (6.2)$$

so that the phases of these couplings must differ by  $\pi$ .

The same argument can be made for RH couplings. Moreover, it is not required that both gauge bosons have LH or RH couplings and the relation in (6.2) assures cancellation of NP contributions to  $\Delta F = 2$  processes for the four possibilities of choosing different couplings. The two scenarios for  $Z'_1$  can be distinguished by rare decays. One can of course also consider L+R scenario but it is not necessary here.

In the case of two gauge bosons with comparable masses also scenarios could be considered in which these bosons have LH and RH couplings of roughly the same size properly tuned to minimise constraints from  $\Delta F = 2$  observables. However, if perturbativity for their couplings is assumed the highest resolution of short distance scales will still be comparable to the one found in the previous section. On the other hand with two gauge bosons having LH and RH couplings of the same size, the correlations between  $\Delta F = 1$  observables could be modified with respect to the ones presented in our paper. We will return to this possibility in the future.

### 6.3 The case of a degenerate scalar and pseudo-scalar pair

We proceed to consider a model consisting of a scalar  $H^0$  and a pseudo-scalar  $A^0$  with equal (or nearly degenerate) mass  $M_{H^0} = M_{A^0} = M_H$ . This is, for example, essentially realised for 2HDMs in a decoupling regime, where  $H^0$  and  $A^0$  are much heavier than the SM Higgs  $h^0$  and almost degenerate in mass. Allowing for a scalar  $H^0$  and pseudo-scalar  $A^0$  with equal couplings to quarks, i.e.

$$\mathcal{L} \ni \bar{D}_L \tilde{\Delta} D_R (H^0 + iA^0) + \text{h.c.}, \quad (6.3)$$

where  $D = (d, s, b)$  and  $\tilde{\Delta}$  is a matrix in flavour space, gives the couplings

$$\Delta_R^{qb}(H^0) = \tilde{\Delta}^{qb}, \quad \Delta_R^{qb}(A^0) = i\tilde{\Delta}^{qb}, \quad \Delta_L^{qb}(H^0) = (\tilde{\Delta}^{bq})^*, \quad \Delta_L^{qb}(A^0) = -i(\tilde{\Delta}^{bq})^*. \quad (6.4)$$

Restricting the couplings to be purely LH or RH and assuming a degenerate mass, we see from inspection of (5.1) that the contributions to  $(M_{12}^*)_{H}^{bq}$  will automatically cancel, without fine-tuning in the couplings. However, if both LH and RH couplings are present, the LR operator will contribute to the mixing. In 2HDMs with MFV, for example, the  $\Delta_L^{qb}$  couplings are suppressed by  $m_q/m_b$  relative to  $\Delta_R^{qb}$ , which will give small but non-zero contributions to the mixing even in the limit of a degenerate heavy neutral scalar and pseudoscalar.

Let us consider the case of only LH (or RH) couplings, and set  $|\Delta_L^{sb}| = |\Delta_P^{\mu\mu}| = 3.0$  as before. Then, for observable deviations in  $\bar{R}_s$  greater than 30%, we find that this model can probe scales up to  $M_H = 850$  TeV, which is comparable to the two scenarios discussed in sections 5.3 and 5.4.

#### 6.4 GIM case

If there are no FCNCs at the tree-level, then new particles entering various box and penguin diagrams enter the game, making the correlations between  $\Delta F = 1$  and  $\Delta F = 2$  processes more difficult to analyse. However, it is evident that for the same couplings NP effects in this case will be significantly suppressed relative to the scenarios discussed until now. This is good for suppressing NP contributions at relative low scales but it does not allow us to reach energy scales as high as in the case of FCNCs at the tree level.

Assuming that the involved one-loop functions are  $\mathcal{O}(1)$  and comparing tree-level expressions for  $\Delta F = 2$  and  $\Delta F = 1$  effective Hamiltonians with those one would typically get by calculating box and penguin diagrams we find that NP contributions from loop diagrams are suppressed relative to tree diagrams by the additional factors

$$\kappa(\Delta F = 2) = \frac{\Delta_{L,R}^2}{32\pi^2}, \quad \kappa(\Delta F = 1) = \frac{\Delta_{L,R}^2}{8\pi^2}. \quad (6.5)$$

For couplings  $\Delta_{L,R} \approx 3$  these suppressions amount approximately to 1/40 and 1/10 respectively. This in turn implies that at the same precision as in the previous sections the maximal scales at which NP could be studied are reduced by roughly factors of 6 and 3 for  $\Delta F = 2$  and  $\Delta F = 1$ , respectively. For smaller couplings this reduction is larger. Detail numbers are not possible without the study of a concrete model.

### 7 Can we determine $M_{Z'}$ beyond the LHC scales?

We have seen that all observables considered in  $Z'$  scenarios depend on the ratios of the  $Z'$  couplings over the  $Z'$  mass  $M_{Z'}$  as listed in (2.1) and (2.2). By assuming the largest couplings consistent with perturbativity we have succeeded to give an idea about the highest values of  $M_{Z'}$  that could still allow us to study the structure of the NP involved. However it is not guaranteed that the  $Z'$  couplings are that large and  $M_{Z'}$  could also be smaller, yet still significantly higher than the LHC scales.

Let us therefore assume that in the future all observables considered in our paper have been measured with high precision and all CKM and hadronic uncertainties have been reduced to a few percent level. Moreover, let us assume significant departures from

SM predictions have been identified with the pattern of deviations from the SM pointing towards the existence of a heavy  $Z'$ . We then ask the question whether in this situation we could determine at least approximately the value of  $M_{Z'}$  on the basis of flavour observables.

Before we answer this question let us recall that the masses of the SM gauge bosons  $Z$  and  $W^\pm$  were predicted in the 1970's, several years before their discovery, due to the knowledge of  $G_F$ ,  $\alpha_{\text{em}}$  and  $\sin^2 \theta_W$  - all determined in low energy processes. Similarly also the masses of the charm quark and top quark could be approximately predicted. Yet, this was only possible because it was done within a concrete theory, the SM, which allowed one to use all measured low energy processes at that time. Thus within a specific theory with not too many free parameters, one could imagine that also the mass of  $Z'$  could be indirectly determined. But what if the only information about  $Z'$  comes from the processes considered by us?

Here we would like to point out the possibility of determining  $M_{Z'}$  from flavour observables provided the next  $e^+e^-$  or  $\mu^+\mu^-$  collider, still with center of mass energies well below  $M_{Z'}$ , could determine indirectly the leptonic ratios in (2.2). This will only be possible if in these collisions some departures from SM expectations will also be found. Only the determination of the ratios involving muon couplings is necessary as the one involving neutrino couplings could be obtained through the  $SU(2)_L$  relation in (2.2). These ratios could of course be obtained from the upgraded LHC, but the presence of protons in the initial state will complicate this determination.

Knowing the values of the ratios in (2.2), one could determine all ratios in (2.1) through rare  $K$  and  $B_{s,d}$  decays. Here the decays governed by  $b \rightarrow s$  transitions are superior to the other decays as there are many of them, yet if the decays  $K_L \rightarrow \pi^0 \ell^+ \ell^-$  could be measured and the hadronic matrix elements entering  $\varepsilon'/\varepsilon$  brought under control also the  $K$  system would be of interest here.

What is crucial for the idea that follows is that  $\Delta F = 2$  transitions have not yet been used for the determination of the ratios in (2.1), that both LH and RH are present, and that both are relevant for rare decays to the extent that the ratios in (2.1) can be measured. This would not allow the resolution of the highest scales but would still provide interesting results.

Now for the main point. With the ratios in (2.1) determined by rare decays, the dependence of the right-hand sides of (4.1) and (4.2) on  $M_{Z'}$  is only through the hadronic matrix elements of the involved operators. Although this dependence, as given in table 5, is only logarithmic, it is sufficiently strong in the presence of LR operators to allow one to estimate the value of  $M_{Z'}$  with the help of  $\Delta F = 2$  observables. To this end precise knowledge of the relevant hadronic matrix elements is necessary. This also applies to CKM parameters entering SM contributions. In principle the same discussions can be made for scalars but it is unlikely that they can play a prominent role in  $e^+e^-$  collisions.

## 8 Conclusions

In this paper we have addressed the question of whether we could learn something about the very short distance scales that are beyond the reach of the LHC on the basis of quark

flavour observables alone. Certainly this depends on the size of NP, its nature and in particular on the available precision of the SM predictions for flavour observables. The latter precision depends on the extraction of CKM parameters from the data and on the theoretical uncertainties. Both are expected to be reduced in this decade down to 1–2%, which should allow NP to be identified even if it contributed only at the level of 10–30% to the branching ratios.

Answering this question in the context of  $Z'$  models and assuming that all its couplings to SM fermions take values of at most 3.0, our main findings are as follows:

- $\Delta F = 2$  processes alone cannot give us any concrete information about the nature of NP at short distance scales beyond the reach of the LHC. In particular if some deviations from SM expectations will be observed, it will not be possible to find out whether they come from LH currents, RH currents or both.
- On the other hand future precise measurements of several  $\Delta F = 1$  observables and in particular correlations between them can distinguish between LH and RH currents, but the maximal resolution consistent with perturbativity strongly depends on whether only LH or only RH or both LH and RH flavour changing  $Z'$  couplings to quarks are present in nature.
- If only LH or RH couplings are present in nature we can in principle reach scales of 200 TeV and 15 TeV for  $K$  and  $B_{s,d}$ , respectively. These numbers depend on the room left for NP in  $\Delta F = 2$  observables, which have an important impact on the resolution available in these NP scenarios.
- Smaller distance scales can only be resolved if both RH and LH couplings are present in order to cancel the NP effects on the  $\Delta F = 2$  observables. Moreover, to achieve the necessary tuning, the couplings should differ considerably from each other. This large hierarchy of couplings is dictated primarily by the ratio of hadronic matrix elements of LR  $\Delta F = 2$  operators and those for LL and RR operators and by the room left for NP in  $\Delta F = 2$  processes. We find that in this case the scales as high as 2000 TeV and 160 TeV for  $K$  and  $B_{s,d}$  systems, respectively, could be in principle resolved.
- A study of tree-level (pseudo-)scalar exchanges shows that  $B_{s,d} \rightarrow \mu^+ \mu^-$  can probe scales close to 1000 TeV, both for scenarios with purely LH or RH scalar couplings to quarks and for scenarios allowing for both LH and RH couplings. For the limit of a degenerate scalar and pseudoscalar NP effects in  $\Delta F = 2$  observables can cancel even without imposing a tuning on the couplings.
- We have discussed models with several gauge bosons. Also in this case the basic strategy for being able to explore very high energy scales is to break the stringent correlation between  $\Delta F = 1$  and  $\Delta F = 2$  processes and to suppress NP contributions to the latter without suppressing NP contributions to rare decays. The presence of a second heavy neutral gauge boson allows us to achieve the goal with only LH or RH currents by applying an appropriate tuning.

- While the highest achievable resolution in the presence of several gauge bosons is comparable to the case of a single gauge boson because of the perturbativity bound, the correlations between  $\Delta F = 1$  observables could differ from the ones presented here. This would be in particular the case if LH and RH couplings of these bosons were of similar size. A detailed study of such scenarios would require the formulation of concrete models.
- If FCNCs only occur at one loop level the highest energy scales that can be resolved for maximal couplings are typically reduced relative to the case of tree-level FCNCs by a factor of at least 3 and 6 for  $\Delta F = 1$  and  $\Delta F = 2$  processes, respectively.
- We have also presented a simple idea for a rough indirect determination of  $M_{Z'}$  by means of the next linear  $e^+e^-$  or  $\mu^+\mu^-$  collider and precision flavour data. It uses the fact that the LR operators present in  $\Delta F = 2$  transitions have large anomalous dimensions so that  $M_{Z'}$  can be determined through renormalisation group effects provided it is well above the LHC scales.

In summary we have demonstrated that NP with a particular pattern of dynamics could be investigated through rare  $K$  and  $B_{s,d}$  decays even if the scale of this NP would be close to the Zeptouniverse. As expected from other studies it is in principle easier to reach the Zeptouniverse with the help of rare  $K$  decays than  $B_{s,d}$  decays. However, this assumes the same maximal couplings in these three systems and this could be not the case. Moreover, in the presence of tree-level pseudoscalar exchanges very short distance scales can be probed by  $B_{s,d} \rightarrow \mu^+\mu^-$  decays.

We should also emphasise that although our main goal was to reach the highest energy scales with the help of rare decays, it will of course be exciting to explore any scale of NP above the LHC scales in this decade. Moreover, we still hope that high energy proton-proton collisions at the LHC will exhibit at least some foot prints of new particles and forces. This would greatly facilitate flavour analyses as the one presented here.

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## A $\Delta F = 1$ master functions

Here we collect the  $\Delta F = 1$  functions that enter the various rare  $K$  and  $B_{s,d}$  decays discussed in this paper. We do not give the more complicated expressions for NP contributions to  $\Delta F = 2$  observables. They can be found in section 3.2.1 of [16]. Note that we have updated the relevant hadronic matrix elements, as given in table 5.

The master functions in question that enter our analysis are given as follows [16]:

$$X_L(K) = \eta_X X_0(x_t) + \frac{\Delta_L^{\nu\bar{\nu}}}{g_{\text{SM}}^2 M_{Z'}^2} \frac{\Delta_L^{sd}}{V_{ts}^* V_{td}}, \quad (\text{A.1})$$

$$X_R(K) = \frac{\Delta_L^{\nu\bar{\nu}}}{g_{\text{SM}}^2 M_{Z'}^2} \frac{\Delta_R^{sd}}{V_{ts}^* V_{td}}, \quad (\text{A.2})$$

$$X_L(B_q) = \eta_X X_0(x_t) + \left[ \frac{\Delta_L^{\nu\nu}}{M_{Z'}^2 g_{\text{SM}}^2} \right] \frac{\Delta_L^{qb}}{V_{tq}^* V_{tb}}, \quad (\text{A.3})$$

$$X_R(B_q) = \left[ \frac{\Delta_L^{\nu\nu}}{M_{Z'}^2 g_{\text{SM}}^2} \right] \frac{\Delta_R^{qb}}{V_{tq}^* V_{tb}}, \quad (\text{A.4})$$

$$Y_A(K) = \eta_Y Y_0(x_t) + \frac{[\Delta_A^{\mu\bar{\mu}}]}{M_{Z'}^2 g_{\text{SM}}^2} \left[ \frac{\Delta_L^{sd} - \Delta_R^{sd}}{V_{ts}^* V_{td}} \right] \equiv |Y_A(K)| e^{i\theta_Y^K}, \quad (\text{A.5})$$

$$Y_A(B_q) = \eta_Y Y_0(x_t) + \frac{[\Delta_A^{\mu\bar{\mu}}]}{M_{Z'}^2 g_{\text{SM}}^2} \left[ \frac{\Delta_L^{qb} - \Delta_R^{qb}}{V_{tq}^* V_{tb}} \right] \equiv |Y_A(B_q)| e^{i\theta_Y^{B_q}}. \quad (\text{A.6})$$

Here  $\eta_{X,Y}$  are factors which include both QCD corrections [97–99] and NLO electroweak correction [100–102]. For  $m_t = m_t(m_t)$  they are close to unity ,

$$\eta_X = 0.994, \quad \eta_Y = 0.9982. \quad (\text{A.7})$$

$g_{\text{SM}}$  is defined in (3.6). Explicit expressions for the SM functions  $X_0(x_t)$  and  $Y_0(x_t)$  can be found in [1].

## B Basic formulae for observables

### B.1 Operators

We list here the operators that contribute to  $\Delta F = 2$  observables. Specifically, for the  $B_q^0 - \bar{B}_q^0$  system the full basis is given as follows [1, 88, 89]:

$$Q_1^{\text{VLL}} = (\bar{b}\gamma_\mu P_L q) (\bar{b}\gamma^\mu P_L q), \quad Q_1^{\text{VRR}} = (\bar{b}\gamma_\mu P_R q) (\bar{b}\gamma^\mu P_R q), \quad (\text{B.1})$$

$$Q_1^{\text{LR}} = (\bar{b}\gamma_\mu P_L q) (\bar{b}\gamma^\mu P_R q), \quad Q_2^{\text{LR}} = (\bar{b}P_L q) (\bar{b}P_R q), \quad (\text{B.2})$$

$$Q_1^{\text{SLL}} = (\bar{b}P_L q) (\bar{b}P_L q), \quad Q_1^{\text{SRR}} = (\bar{b}P_R q) (\bar{b}P_R q), \quad (\text{B.3})$$

$$Q_2^{\text{SLL}} = (\bar{b}\sigma_{\mu\nu} P_L q) (\bar{b}\sigma^{\mu\nu} P_L q), \quad Q_2^{\text{SRR}} = (\bar{b}\sigma_{\mu\nu} P_R q) (\bar{b}\sigma^{\mu\nu} P_R q), \quad (\text{B.4})$$

where  $P_{R,L} = (1 \pm \gamma_5)/2$ . Colour indices are suppressed as they are summed up in each factor. For  $K^0 - \bar{K}^0$  mixing  $b \leftrightarrow s$  and  $q \leftrightarrow d$  have to be interchanged.

### B.2 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The branching ratios for these two modes can be written in the general form

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ \left[ \left( \frac{\text{Im} X_{\text{eff}}}{\lambda^5} \right)^2 + \left( \frac{\text{Re} X_{\text{eff}}}{\lambda^5} - \bar{P}_c(X) \right)^2 \right], \quad (\text{B.5})$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{\text{Im} X_{\text{eff}}}{\lambda^5} \right)^2, \quad (\text{B.6})$$

with the Cabibbo angle  $\lambda = 0.2252(9)$  and where [103]

$$\kappa_+ = (5.21 \pm 0.025) \cdot 10^{-11} \left( \frac{\lambda}{0.2252} \right)^8, \quad \kappa_L = (2.25 \pm 0.01) \cdot 10^{-10} \left( \frac{\lambda}{0.2252} \right)^8, \quad (\text{B.7})$$

and [103–107]

$$\bar{P}_c(X) = \left( 1 - \frac{\lambda^2}{2} \right) P_c(X), \quad P_c(X) = (0.42 \pm 0.03) \left( \frac{0.2252}{\lambda} \right)^4. \quad (\text{B.8})$$

The short distance contributions are described by

$$X_{\text{eff}} = V_{ts}^* V_{td} (X_L(K) + X_R(K)), \quad (\text{B.9})$$

where  $X_{L,R}(K)$  are given in (A.1) and (A.2).

### B.3 $B \rightarrow \{X_s, K, K^*\} \nu \bar{\nu}$

The branching ratios of the  $B \rightarrow \{K, K^*\} \nu \bar{\nu}$  modes in the presence of RH currents can be written as follows [81]

$$\frac{\mathcal{B}(B \rightarrow K \nu \bar{\nu})}{\mathcal{B}(B \rightarrow K \nu \bar{\nu})_{\text{SM}}} = [1 - 2\eta] \epsilon^2, \quad (\text{B.10})$$

$$\frac{\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})}{\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})_{\text{SM}}} = [1 + 1.31\eta] \epsilon^2, \quad (\text{B.11})$$

where

$$\epsilon^2 = \frac{|X_L(B_s)|^2 + |X_R(B_s)|^2}{|\eta_X X_0(x_t)|^2}, \quad \eta = \frac{-\text{Re}(X_L(B_s) X_R^*(B_s))}{|X_L(B_s)|^2 + |X_R(B_s)|^2}, \quad (\text{B.12})$$

with  $X_{L,R}(B_s)$  defined in (A.3) and (A.4).

### B.4 $K_L \rightarrow \mu^+ \mu^-$

Only the so-called short distance (SD) part to a dispersive contribution to  $K_L \rightarrow \mu^+ \mu^-$  can be reliably calculated but it serves as a useful constraint on NP contributing to  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ . It is given by

$$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}} = \kappa_\mu \left( \frac{\text{Re} Y_{\text{eff}}}{\lambda^5} - \bar{P}_c(Y) \right)^2. \quad (\text{B.13})$$

Here

$$\kappa_\mu = (2.01 \pm 0.02) \cdot 10^{-9} \left( \frac{\lambda}{0.2252} \right)^8, \quad Y_{\text{eff}} = V_{ts}^* V_{td} Y_A(K), \quad (\text{B.14})$$

where  $Y_A(K)$  is given in (A.5) and

$$\bar{P}_c(Y) = \left( 1 - \frac{\lambda^2}{2} \right) P_c(Y), \quad P_c(Y) = (0.115 \pm 0.018) \left( \frac{0.2252}{\lambda} \right)^4, \quad (\text{B.15})$$

with  $P_c(Y)$  at NNLO [108]. The extraction of the short distance part from the data is subject to considerable uncertainties. The most recent estimate gives [109]

$$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}} \leq 2.5 \cdot 10^{-9}, \quad (\text{B.16})$$

to be compared with  $(0.8 \pm 0.1) \cdot 10^{-9}$  in the SM [108].



### B.5 $B \rightarrow K^{(*)} \ell^+ \ell^-$

The effective Hamiltonian for  $b \rightarrow s \ell^+ \ell^-$  transitions, such as  $B \rightarrow K^* \ell^+ \ell^-$ ,  $B \rightarrow K \ell^+ \ell^-$  and  $B \rightarrow X_s \ell^+ \ell^-$ , is given as (see e.g. also [95])

$$H_{\text{eff}}(b \rightarrow s \ell \bar{\ell}) = H_{\text{eff}}(b \rightarrow s \gamma) - \frac{4G_F}{\sqrt{2}} \frac{\alpha}{4\pi} V_{ts}^* V_{tb} \sum_{i=9,10,S,P} [C_i(\mu) Q_i(\mu) + C'_i(\mu) Q'_i(\mu)] \quad (\text{B.17})$$

where

$$Q_9 = (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell), \quad Q'_9 = (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma^\mu \ell), \quad (\text{B.18})$$

$$Q_{10} = (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell), \quad Q'_{10} = (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma^\mu \gamma_5 \ell), \quad (\text{B.19})$$

$$Q_S = m_b (\bar{s} P_R b) (\bar{\ell} \ell), \quad Q'_S = m_b (\bar{s} P_L b) (\bar{\ell} \ell), \quad (\text{B.20})$$

$$Q_P = m_b (\bar{s} P_R b) (\bar{\ell} \gamma_5 \ell), \quad Q'_P = m_b (\bar{s} P_L b) (\bar{\ell} \gamma_5 \ell). \quad (\text{B.21})$$

### B.6 $B_{d,s} \rightarrow \mu^+ \mu^-$

The ratio of the  $B_q \rightarrow \mu^+ \mu^-$  branching ratio, with  $q = s, d$ , relative to its SM estimate is given by [95, 110]

$$\bar{R}_q \equiv \frac{\bar{\mathcal{B}}(B_q \rightarrow \mu^+ \mu^-)}{\bar{\mathcal{B}}(B_q \rightarrow \mu^+ \mu^-)_{\text{SM}}} = \left[ \frac{1 + \mathcal{A}_{\Delta\Gamma}^{\mu\mu}(B_q \rightarrow \mu^+ \mu^-) y_q}{1 + y_q} \right] (|P|^2 + |S|^2), \quad (\text{B.22})$$

where

$$P \equiv \frac{-Y_A(B_q)}{\sin^2 \theta_W C_{10}^{\text{SM}}} + \frac{m_{B_q}^2}{2m_\mu} \frac{m_b}{m_b + m_q} \frac{C_P - C'_P}{C_{10}^{\text{SM}}}, \quad (\text{B.23})$$

$$S \equiv \sqrt{1 - \frac{4m_\mu^2}{m_{B_q}^2} \frac{m_{B_q}^2}{2m_\mu} \frac{m_b}{m_b + m_q} \frac{C_S - C'_S}{C_{10}^{\text{SM}}}}, \quad (\text{B.24})$$

with  $C_{10}^{\text{SM}} = -\eta_Y \sin^{-2} \theta_W Y_0(x_t)$  the only relevant coefficient in the SM and  $Y_A(B_q)$  given in (A.6). The coefficients  $C_P^{(\prime)}$  and  $C_S^{(\prime)}$  correspond to the effective (pseudo-)scalar operators given in (B.21) and (B.20), respectively.  $\mathcal{A}_{\Delta\Gamma}^{\mu\mu}(B_q \rightarrow \mu^+ \mu^-)$  is the mass-eigenstate rate asymmetry for this decay, which is relevant in the case of a non-zero  $B_q$  lifetime difference  $y_q = (\tau_{q,H} - \tau_{q,L})/(\tau_{q,H} + \tau_{q,L})$  [111] i.e. for the  $B_s$  decay where  $y_s$  is non-zero.

In the SM only the heavy mass-eigenstate contributes to these decays, giving a maximal asymmetry,  $\mathcal{A}_{\Delta\Gamma}^{\mu\mu} = 1$ , and thereby a maximal correction to the branching ratio. In  $Z'$  models this asymmetry only differs from its SM value in the presence of new CP violating phases, whereas in models with new scalars it can deviate from one also in the absence of such phases [94, 110].

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